Galois Theory

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Abstract

Galois Theory is a section of Abstract Algebra that brings together concepts from both group theory and ring theory. Specifically it connects finite, separable, and normal field extensions with a group that is made up of automorphisims of a given field. This area of mathematics focuses on the constructions of Galois groups and Galois extensions. Galois Theory can help us determine the answers as to which regular polygons are constructible with a straight edge and compass and answer questions about finding the roots of polynomials over various fields.

Summary

Galois Group of E over F If $F \subseteq E$ (where F is a field and E is a field extension of F) then the set of automorphisms of E over F is a group under composition of functions. Denoted by $\Gamma(E/F)$.

Galois Extensions For $F \subseteq E$, E is a Galois extension of F then the following statements are equivalent:

i) $F = \Phi(\Gamma(E/F))$.
ii) E is normal and separable over F.
iii) E is the splitting field over F for some separable $f(X) \in F[X]$.