

PRACTICE Comprehensive Exam

Name: _____

Department of Mathematics

Tuesday, March 15, 2016

Closed book. Closed notes. NO CALCULATORS. Time allowed: 3 hours for 5 sections (proportionally less if taking fewer than 5 sections). In other words, 36 minutes for each section taken. Please write very legibly and cross out all scratch work.

Calculus 1

1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Total: _____

Calculus 2

6. _____ 7. _____ 8. _____ 9. _____ 10. _____ Total: _____

Multivariable Calculus

11. _____ 12. _____ 13. _____ 14. _____ 15. _____ Total: _____

Linear Algebra

16. _____ 17. _____ 18. _____ 19. _____ 20. _____ Total: _____

Discrete Mathematics

21. _____ 22. _____ 23. _____ 24. _____ 25. _____ Total: _____

Calculus 1

1. Find values of the constants k and m , if possible, that will make the function f continuous everywhere.

$$f(x) = \begin{cases} x^2 + 5, & x > 2 \\ m(x + 1) + k, & -1 < x \leq 2 \\ 2x^3 + x + 7, & x \leq -1 \end{cases}$$

2. Use the limit laws, and if necessary, L'Hôpital's Rule to find the following limit

$$\lim_{x \rightarrow 0^+} (1 + 2x)^{-3/x}$$

3. Suppose that the number of bacteria in a culture at time t is given by

$$N = 5000(25 + te^{-t/20}).$$

- (a) Find the largest and smallest number of bacteria in the culture during the time interval $0 \leq t \leq 100$.
- (b) At what time during the time interval in part (a) is the number of bacteria decreasing most rapidly?

4. A church window consisting of a rectangle topped by a semicircle is to have a perimeter p . Find the radius of the semicircle if the area of the window is to be maximum.

5. Suppose that you have money in an account that is earning interest at an APR of 4% compounded continuously and that you add a total of \$1000 to the account every year applied at a constant rate so that the rate of change of money M in the account is given by

$$\frac{dM}{dt} = (.04)M + 1000,$$

where time t is measured in years and money M is measured in dollars. Also suppose that you have \$8000 in the account at the start of year three, i.e. $M(3) = 8000$.

- (a) Use a local linear approximation to estimate how much money will be in the account at the end of January of the third year, i.e. use a local linear approximation to estimate $M(3 + \frac{1}{12})$.
- (b) Use the second derivative to determine if your approximation is an overestimate or an underestimate. Explain your answer.