2004: Comprehensive Exam
CALCULUS 1

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3. Suppose that f is a function which satisfies $f(0)=1$, $f'(0)=0$ and $f''(0)=0$. Circle the statements below that could be true of f. Give a possible formula for f(x) for each case you choose.

(a) The graph of f has an inflection point when $x=0$
Ex: $f(x) = x^3+1$

(b) f achieves a local maximum at $x=0$
Ex: $f(x) = -x+1$

(c) f achies a local minimum at $x=0$
Ex: $f(x) = x+1$

d. The graph of is a straight line ~~CANNOT BE TRUE~~

6. Suppose a graphics program is written which displays a circular dot that grows and shrinks periodically. Suppose the radius, as a function of time, satisfies $r(t) = 3\cos(\pi t/4)$. Find the rate at which the area of this circular dot is changing when $t=1$. Show your work.

$A = \pi r^2$

$\frac{d}{dt}(A = \pi(3\cos(\frac{\pi}{4}t))^2)$

$\frac{dA}{dt} = \pi(2(3\cos(\frac{\pi}{4}t)) \cdot -3\sin(\frac{\pi}{4}t) \cdot \frac{\pi}{4}$

$\frac{dA}{dt} = -\frac{9\pi^2}{2}\cos(\frac{\pi}{4}t)\sin(\frac{\pi}{4}t)$

at $t=1$ $\frac{dA}{dt} = \frac{-9\pi^2}{2}\cos(\frac{\pi}{4})\sin(\frac{\pi}{4})$

$= -9\pi^2/2 \cdot (\sqrt{2}/2)(\sqrt{2}/2)$

$= -\frac{9\pi^2}{2}$

≈ -22.21

7. Explain why the function $f(x) = x - \frac{1}{x}$ can have at most one zero with $x<0$ and at most one zero with $x>0$.

This function only has zeros when $x = \frac{1}{x}$ so there is one zero whe $x<0$ at $x=-1$ so $-1 = \frac{1}{-1}$ and one zero when $x>0$ at $x=1$ so $1 = \frac{1}{1}$.

10. Suppose f and g are functions such that $\lim_{x\to 5} f(x)=2$ and $\lim_{x\to 5} g(x)=0$. Of the following statements, only one is a faulty conclusion given the information above. Circle this statement and explain why the statement is not always true.

(a) $\lim_{x\to 5} [f(x)g(x)]$ exists

(b) $\lim_{x\to 5^+} f(x)=2$

(c) $g(5)=0$

(d) The graph of the function $f(x)$ has no vertical asymptote at $x=5$

(c) is not always true because $g(5)$ does not necesarily equal 0. There could be a horizontal asymptote at $x=5$ or a hole at $x=5$.

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1 $f(2) = 2^2 + 3 = 7$

$y = 7x + b$

$3 = 7 \cdot 2 + b$

$b = 11$

$y = 7x + 11$

$y = 7 \cdot 2.3 + 11$
$= 27.1$

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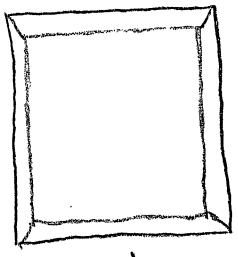
4) The material for the sides of a rectangular picture frame costs 5 cents per inch; the material for the top and bottom of the frame costs 10 cents per inch. Find the dimension of the least expensive frame that will enclose a picture with area 200 in^2. Include an argument that you have really found the minimal cost.



we say the number of inches of 5¢ frame material is x
the number of inches of 10¢ frame material is y

we want $(.05)x = (.1)y$

simplify $y = \frac{1}{2}x$

now we see that when $x = 20$ in $y = 10$ in and

$10in \cdot 20in = 200 in^2$, also $(.05)(20) = (.1)(10)$ ✓

we can see that because the frame material for the sides costs the same as the frame material for the top and bottom, we can assure a minimal cost.

$(.05)(20) = 1$
$(.1)(10) = 1$

so $1+1 = 2$

2$ is our minimum cost for a frame that holds a 200 in^2 picture.

#5)

a) write down the definition of $f'(a)$:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

b) what feature of the graph does $f'(a)$ measure?

$f'(a)$ measures the slope of the tangent line for the function $f(a)$ where the domain of $f'(a)$ is all the possible entries of $f(a)$ where the limit exists, Also the derivitive $F'(a)$ can measure the instantaneous rate of change between a and $f(a)$

c) Explain why your answer in (a) is a reasonable definition for the feature in (b). This should require no more than two to three sentences, or 20-30 words.

we can think of $f'(a)$ as the limit of the slopes of secant lines that intersect our function at the points $f(a)$ and $F(a+h)$, so as $h \to 0$ the slope of our secant lines approaches the slope of our tangent line.

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2 $B(t) = B + .3B - 5$

$B(0) = 5$

3 $f(x) = e^{2x} \sin(\ln x^2)$

$f'(x) = e^{2x} 2 \sin(\ln x^2) + e^{2x} \cos(\ln x^2) \cdot 1/x^2 \cdot 2x$