Multivariable Calculus

Math 224 Spring 2004 | Fowler 112 MWF 2:30pm - 3:25pm

 | http://faculty.oxy.edu/ron/math/224/04/

Class 3: **Monday January 26**

SUMMARY The Dot Product and its Implications and Applications
CURRENT READING Williamson & Trotter, Section 1.4
HOMEWORK Williamson & Trotter, §1.4 #

Dot Product

Given two vectors in $\mathbb{R}^n$, $\vec{x} = (x_1, x_2, \ldots, x_n)$ and $\vec{y} = (y_1, y_2, \ldots, y_n)$ the **dot product** is defined as:

$$\vec{x} \cdot \vec{y} = \sum_{k=1}^{n} x_k y_k = x_1 y_1 + x_2 y_2 + x_3 y_3 + \ldots + x_n y_n$$

The dot product is a very useful operation that allows us to represent a number of interesting results.

Magnitude of a Vector

$$|\vec{x}| = \sqrt{\vec{x} \cdot \vec{x}}$$

Angles Between Vectors

The dot product also defines an expression for the angle between two vector $\vec{x}$ and $\vec{y}$

$$\vec{x} \cdot \vec{y} = |\vec{x}||\vec{y}| \cos(\theta)$$

which leads to the **Cauchy-Schwarz Inequality**

$$\vec{x} \cdot \vec{y} \leq |\vec{x}||\vec{y}|$$

Law of Cosines

$$|\vec{x} - \vec{y}|^2 = |\vec{x}|^2 + |\vec{y}|^2 - 2|\vec{x}||\vec{y}| \cos(\theta)$$

Triangle Inequality

$$|\vec{x} + \vec{y}| \leq |\vec{x}| + |\vec{y}|$$

Properties of the Dot Poduct

Positivity: $\vec{x} \cdot \vec{x} > 0$ (except when $\vec{x} = \vec{0}$)
Symmetry: $\vec{x} \cdot \vec{y} = \vec{y} \cdot \vec{x}$
Additivity: $(\vec{x} + \vec{y}) \cdot \vec{z} = \vec{x} \cdot \vec{z} + \vec{y} \cdot \vec{z}$
Homogeneity: $(r\vec{x}) \cdot \vec{y} = r(\vec{x} \cdot \vec{y})$

GROUPWORK

For the given vector $\vec{u} = (3, 1, 1)$ and $\vec{v} = (4, 1, 0)$ find $\vec{u} \cdot \vec{v}$, $|\vec{u}|$, $|\vec{v}|$ the angle between $\vec{v}$ and $\vec{u}$ and normalize each of the vectors.

EXERCISE

Williamson & Trotter, page 32, # 28. Show that the sum of the squares of the lengths of a the four sides of a parallelogram is equal to the sum of the squares of the diagonals. [HINT: Sketch the two vectors $\vec{x}$ and $\vec{y}$ and obtain expressions for the diagonals in terms of $\vec{x}$ and $\vec{y}$.]