Test 2: Linear Systems

Math 214 Spring 2007 — Friday April 20

 — 2:30pm-3:25pm

Name: ______________________________

Directions: Read *all* problems first before answering any of them. There are 7 pages in this test (including this page). This is a 55-minute, no-notes, closed book, test. **No calculators.** You must show all relevant work to support your answers. Use complete English sentences and CLEARLY indicate your final answers to be graded from your "scratch work."

You may not discuss the questions on this test with any other student.

Pledge: I, ______________________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

No.	Score	Maximum
1		20
2		20
3		20
4		20
5		20
BONUS		10
Total		**100**

1. **Column Space, Linear Combinations, Solvability.** (*20 points.*)

(a) Give the definition of the phrase "the column space of a matrix."

(b) Prove or give a counter-example: IF A is an $m \times n$ matrix and $\vec{b}$ is a vector in $\mathbb{R}^m$ such that the equation $A\vec{x} = \vec{b}$ has one or more solutions, THEN $\vec{b}$ is a linear combination of the columns of A.

2. **Basis, Vector Space.** (*20 points.*)

(a) Give the definition of the phrase "a basis for a vector space."

(b) Prove or give a counter-example: IF $\vec{u}$, $\vec{v}$ and $\vec{w}$ are unit vectors in $\mathbb{R}^3$ such that none of them is a multiple of another, THEN $\{\vec{u}, \vec{v}, \vec{w}\}$ is a basis for $\mathbb{R}^3$.

3. **Eigenvector, Eigenvalue.** (*20 points.*)

(a) Give the definition of the phrase "an eigenvector of a matrix."

(b) Prove or give a counter-example: IF $\vec{v}$ and $\vec{w}$ are eigenvectors of a matrix A such that $\vec{v}$ and $\vec{w}$ have the same eigenvalue λ, THEN $\vec{v} + \vec{w}$ is an eigenvector of A.

4. **Subspace, Dimensional, Orthogonal Complements.** (*20 points.*)

(a) Give the definition of the phrase "a subspace of a vector space."

(b) Let $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ be linearly independent vectors in $\mathbb{R}^3$ and let $\mathcal{W}$ be the orthogonal complement of $\text{span}(\vec{v}, \vec{w})$. Is $\mathcal{W}$ a subspace of $\mathbb{R}^3$? If it is a subspace of $\mathbb{R}^3$, give the dimension of $\mathcal{W}$ and explain how you find its dimension. If $\mathcal{W}$ is not a subspace of $\mathbb{R}^3$, explain why it is not.

5. **Linearly Independence, Invertibility.** (*20 points.*)

(a) Give the definition of the phrase "a linearly independent set of vectors in $\mathbb{R}^n$." (You CAN NOT just say "Not Linearly Dependent!")

(b) IF an $n \times n$ matrix A has n linearly independent columns, THEN (Put a CHECK-MARK $\surd$ in the box next to each of the statements below that is true):

- ☐ The rows of A are linearly independent
- ☐ The column space of A is $\mathbb{R}^n$
- ☐ Every row of A is a linear combination of the columns of A
- ☐ The reduced row echelon form of A is the identity matrix
- ☐ $\det(A) = 0$
- ☐ For every vector $\vec{b}$ in $\mathbb{R}^n$, the equation $A\vec{x} = \vec{b}$ has exactly one solution
- ☐ The null space of A contains no vectors other than the $\vec{0}$ vector
- ☐ A is a non-singular matrix
- ☐ $\text{rank}(A) = n$
- ☐ A^T is invertible.

No explanation needs to be given for why you think your chosen statements are true.

BONUS QUESTION. Linearly Independence, Invertibility. (*10 points.*)
Provide detailed written explanations for each of the statements in Question 5, explaining why the statement is either TRUE or FALSE.