Linear Systems

Math 214 Spring 2007 Fowler 110 MWF 2:30pm - 3:25pm

http://faculty.oxy.edu/ron/math/214/07/

Class 21: **Friday March 23**

TITLE Introducton to Eigenvectors and Eigenvalues
CURRENT READING Poole 4.1

Summary
Let's explore the wonderful world of *eigenvectors*, *eigenvalues* and *eigenspaces* of a square 2x2 matrix.

Homework Assignment
HW #19 Poole, Section 4.1: **4,5**,*6*,**10,11**,*16,17*,**21,22**. *EXTRA CREDIT 36, 37*

DEFINITION

An **eigenvalue** of a $n \times n$ matrix A is a scalar value λ such that there exists a non-zero vector $\vec{x}$ where $A\vec{x} = \lambda\vec{x}$. The vector $\vec{x}$ is called the **eigenvector** corresponding to the **eigenvalue** λ.

1. Eigenvalues and Eigenvectors

Interestingly, in order to find the eigenvalues of a matrix, one just has to solve the equation $A\vec{x} - \lambda\vec{x} = \vec{0}$ or $(A - \lambda I)\vec{x} = \vec{0}$.

This means that the eigenvectors of matrix A corresponding to eigenvalue λ lie in the nullspace of the matrix $A - \lambda I$. It's not clear right now, but it turns out that the eigenvalues of A are the solution of the equation $\det(A - \lambda I) = 0$. This equation is known as the **characteristic polynomial** of the matrix A.

EXAMPLE

Find the eigenvalues and eigenvectors of $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

2. Eigenvectors and Eigenspace

DEFINITION

Given a $n \times n$ matrix A with eigenvalue λ the set of all vectors corresponding to the eigenvalue λ plus the zero vector is called the **eigenspace** of λ and is denoted E_λ.

Exercise

Write down the eigenspaces associated with the eigenvalues of $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

EXAMPLE

Consider the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Let's show that the eigenvalues of A are the solution of $\lambda^2 - \mathrm{tr}(A)\lambda + \det(A) = 0$.

3. Properties of the Eigenvalues of a Matrix

The **Product** of the eigenvalues equals the determinant of the matrix.

$$\lambda_1 \lambda_2 = |A|$$

The **Sum** of the eigenvalues equals the trace of the matrix (the sum of the diagonal entries)

$$\lambda_1 + \lambda_2 = \sum_{i=1}^{2} A_{ii}$$