Math 214 Spring 2007 **SHOW ALL YOUR WORK** Quiz **Six**

**1.** Consider the system of equations $\begin{bmatrix} b & 3 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$ where $b$ is an unknown parameter.

**a.** *(4 points).* If $b = 2$ what are the column space and the nullspace of the coefficient matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$? Write expressions describing these subspaces. Also give the rank of $A$, the dimension of the nullspace and the dimension of the column space.

$b=2$ $\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} \to \begin{pmatrix} 2 & 3 \\ 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 3/2 \\ 0 & 0 \end{pmatrix} = \text{rref}(A)$

$\text{rank}(A) = 1 = \dim \text{col}(A)$

$\text{col}(A) = \text{span}\left\{\begin{pmatrix} 2 \\ 4 \end{pmatrix}\right\}$

$\text{rank} + \dim(\text{null}(A)) = \#$ of columns

$1 + \text{nullity} = 2$

$\text{nullity} = 2-1 = 1$

$\text{null}(A) = \text{span}\left\{\begin{pmatrix} -3/2 \\ 1 \end{pmatrix}\right\}$

= line thru the origin $2x+3y=0$

col A = line through the origin $2x - y = 0$

**b.** *(4 points).* If $b = 1$ how do the column space and nullspace of the coefficient matrix $A = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$ change from your answer in **(a)**? Write down the rank of $A$, the dimension of the nullspace and the dimension of the column space.

$b=1$ $\begin{pmatrix} 1 & 3 \\ 4 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & 3 \\ 0 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \text{rref}(A)$

$\text{rank}(A) = 2 = \dim \text{col}(A) \Rightarrow \text{span}\left\{\begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 6 \end{pmatrix}\right\} = \text{col}(A) = \mathbb{R}^2$

$\dim(\text{null}(A)) = \#$ of columns $-$ rank

$= 2 - 2 = 0$

$\text{null}(A) = \{\vec{0}\}$

**c.** *(2 points).* How does the rank of the coefficient matrix $\begin{bmatrix} b & 3 \\ 4 & 6 \end{bmatrix}$ depend on the value of $b$? Use this information to determine for what values of $b$ the system has 1 unique solution and explain how you know the solution will be unique for these values of $b$.

$b = 2$, $\text{rank}(A) = 1 \Rightarrow \dim \text{col}(A) = 1$

$b \neq 2$, $\text{rank}(A) = 2 \Rightarrow \dim \text{col}(A) = 2 \Rightarrow \text{col}(A) = \mathbb{R}^2$

when $b \neq 2$, $\text{col}(A) = \mathbb{R}^2$ so the linear system $A\vec{x} = \vec{y}$ must have a unique solution since $\vec{y} \in \text{col}(A) = \mathbb{R}^2$

when $b \neq 2$, $\text{rref}(A) = I$ so $A^{-1}$ exists, so $\vec{x} = A^{-1}\vec{y}$ is a unique solution which exists