Math 214 Spring 2007

Quiz **10** **Linear Systems**

Name: ______________________________

Date: ____________________ **Friday April 13**
Time Begun: ____________________ Ron Buckmire
Time Ended: ____________________

Topic : Orthogonality and Orthgogonal Complements

The idea behind this quiz is for you to indicate your understanding of orthogonality and orthogonal complements.

Reality Check:

EXPECTED SCORE : __________**/10** ACTUAL SCORE : __________**/10**

Instructions:

1. Please look for a hint on this quiz posted to `faculty.oxy.edu/ron/math/214/07/`
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday April 16**, in class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Goal: To project $\vec{v} = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}$ onto the orthogonal complement of the plane $x - y + 3z = 0$ (denoted by $\mathcal{W}$) in $\mathbb{R}^3$.

(a) (*2 points*). If the set of points (x, y, z) in $\mathbb{R}^3$ are represented as the vector $\vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, the set of points on the plane $x - y + 3z = 0$ can be represented as the vectors found in the nullspace of what matrix? **(Re-write $x - y + 3z = 0$ as $A\vec{x} = 0$ and identify A and list its dimensions)**.

(b) (*2 points*). Since the plane $x - y + 3z = 0$ is a 2-dimensional object and given that the rank of the matrix A is 1, write down a basis for null(A) which contains 2 vectors.

(c) (*2 points*). Using the Rank Theorem, write down a basis for the orthogonal complement of the nullspace of A. **(HINT: what is the dimension of this orthogonal complement?)**

(d) (*2 points*). Your answer in **(c)** is a basis for the orthogonal complement of the nullspace of A, in other words $\mathcal{W}^{\perp}$. Find $\vec{w}_1 = \text{proj}_{\mathcal{W}^{\perp}}(\vec{v})$, where $\vec{v} = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}$.

(e) (*2 points*). Use your answer to find $\vec{w}_2 = \text{proj}_{\mathcal{W}}(\vec{v})$. What (two) properties do $\vec{w}_1$ and $\vec{w}_2$ have that you can check to confirm your values of $\vec{w}_1$ and $\vec{w}_2$ are correct?