Math 214 Spring 2006 **SHOW ALL YOUR WORK** Quiz Ten

Goal: To project $\vec{v} = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}$ onto the orthogonal complement of the plane $x - y + 3z = 0$ (denoted by $\mathcal{W}$) in $\mathbb{R}^3$.

(a) (*2 points*). If the set of points (x, y, z) in $\mathbb{R}^3$ are represented as the vector $\vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, the set of points on the plane $x - y + 3z = 0$ can be represented as the vectors found in the nullspace of what matrix? (**Re-write** $x - y + 3z = 0$ **as** $A\vec{x} = 0$ **and identify** A **and list its dimensions**).

$$\begin{bmatrix} 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

A is 1 x 3 (1 row, 3 cols)

$A = \begin{bmatrix} 1 & -1 & 3 \end{bmatrix}$

(b) (*2 points*). Since the plane $x - y + 3z = 0$ is a 2-dimensional object and given that the rank of the matrix A is 1, write down a basis for null(A) which contains 2 vectors.

$$\text{null}(A) = \begin{pmatrix} -3z + y \\ y \\ z \end{pmatrix} = y \begin{pmatrix} +1 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} = \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \right\}$$

(c) (*2 points*). Using the Rank Theorem, write down a basis for the orthogonal complement of the nullspace of A. (**HINT: what is the dimension of this orthogonal complement?**)

The orthogonal complement must be orthogonal to null(A), i.e. row(A).

$$(\text{null}(A))^{\perp} = \text{span}\left\{ \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \right\} = \text{row}(A)$$

(d) (*2 points*). Your answer in (c) is a basis for the orthogonal complement of the nullspace of A, in other words $\mathcal{W}^{\perp}$. Find $\vec{w}_1 = \text{proj}_{\mathcal{W}^{\perp}}(\vec{v})$

$$\vec{w}_1 = \frac{\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix}}{\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}} \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = \frac{-4}{11} \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -4/11 \\ 4/11 \\ -12/11 \end{pmatrix}$$

(d) (*2 points*). Use your answer to find $\vec{w}_2 = \text{proj}_{\mathcal{W}}(\vec{v})$. What properties do $\vec{w}_1$ and $\vec{w}_2$ have that you can check to confirm your answers?

$\vec{w}_2 + \vec{w}_1 = \vec{v}$ and $\vec{w}_1 \cdot \vec{w}_2 = 0$

$$\vec{w}_2 = \vec{v} - \vec{w}_1 = \begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix} - \begin{pmatrix} -4/11 \\ 4/11 \\ -12/11 \end{pmatrix} = \begin{pmatrix} 15/11 \\ 51/11 \\ 12/11 \end{pmatrix}$$

$$\begin{pmatrix} 15/11 \\ 51/11 \\ 12/11 \end{pmatrix} \cdot \begin{pmatrix} -4/11 \\ 4/11 \\ -12/11 \end{pmatrix} = \frac{-60 + 204 - 144}{121} = 0$$