Math 214 Spring 2007

Quiz 9 — **Linear Systems**

Name: ______________________________

Date: ____________________ **Friday April 6**
Time Begun: ____________________ Ron Buckmire
Time Ended: ____________________

Topic : Diagonalization of a Matrix and its Implications for Matrix Exponentiation

The idea behind this quiz is for you to indicate your understanding of the application of finding eigenvalues and eigenvectors associated with a matrix to computing steady state asymptotic values of the matrix.

Reality Check:

EXPECTED SCORE : __________**/10** ACTUAL SCORE : __________**/10**

Instructions:

1. Please look for a hint on this quiz posted to `faculty.oxy.edu/ron/math/214/07/`
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday April 9**, in class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Math 214 Spring 2007 **SHOW ALL YOUR WORK** Quiz **Nine**

1. Consider the matrix $A = \begin{bmatrix} 1/2 & 1/2 \\ 1 & 0 \end{bmatrix}$. We want to obtain a value for $A^\infty = \lim_{n\to\infty} A^n$.

a. (*4 points*). Find the eigenvalues and eigenvectors of A.

b. (*2 points*) Show that $AS = S\Lambda$ or $A = S\Lambda S^{-1}$, where the columns of S are formed by the eigenvectors of A and Λ is a diagonal matrix with the eigenvalues of A along the diagonal and zeroes elsewhere.

c. (*2 points*). Compute $A^n = S\Lambda^n S^{-1}$.

d. (*2 points*). Use your answer from **c** to show that $A^\infty = \begin{bmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$.