Math 214 Spring 2006 **SHOW ALL YOUR WORK** **BONUS** Quiz 9

1. *10 points.* **Poole, page 362, #9.** Let $A = \begin{bmatrix} -5 & -6 & 3 \\ 3 & 4 & -3 \\ 0 & 0 & -2 \end{bmatrix}$

a. *3 points.* Show that the characteristic polynomial is $p(\lambda) = 4 - 3\lambda^2 - \lambda^3$.

$$\det(A - \lambda I) = \begin{vmatrix} -5-\lambda & -6 & 3 \\ 3 & 4-\lambda & -3 \\ 0 & 0 & -2-\lambda \end{vmatrix} = (-2-\lambda)\begin{vmatrix} -5-\lambda & -6 \\ 3 & 4-\lambda \end{vmatrix}$$

$$= (-2-\lambda)\left[(-5-\lambda)(4-\lambda) + 18\right]$$

$$= -(\lambda+2)\left[-20 - 4\lambda + \lambda^2 + 5\lambda + 18\right] = -(\lambda+2)(\lambda^2+\lambda-2)$$

$$= -\left[\lambda^3 + \lambda^2 - 2\lambda + 2\lambda^2 + 2\lambda - 4\right] = -\lambda^3 - 3\lambda^2 + 4 = 0$$

b. *1 points.* Find all the eigenvalues of A

$$p(1) = -1 - 3 + 4 = 0$$

$$p(-2) = -(-8) - 3 \cdot 4 + 4 = 0$$

$$p(\lambda) = -(\lambda+2)(\lambda-1)(\lambda+2) = (\lambda+2)^2(1-\lambda).$$

$$\operatorname{eig}(A) = -2, -2, 1$$

c. *3 points.* Find bases for each of the eigenspaces of A

$$E_{-2} = \operatorname{null}(A + 2I)$$

$$(A + 2I \,|\, 0) = \begin{vmatrix} -3 & -6 & 3 \\ 3 & 6 & -3 \\ 0 & 0 & 0 \end{vmatrix} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$x + 2y - z = 0$

y any

x any

$$\begin{pmatrix} x \\ y \\ x+2y \end{pmatrix} = E_{-2}$$

$$x\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} y = E_{-2}$$

$$(A - I ; 0) = \begin{vmatrix} -6 & -6 & 3 \\ 3 & 3 & -3 \\ 0 & 0 & -3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix}$$

$x + y = 0$

$z = 0$

$$\begin{pmatrix} x \\ -x \\ 0 \end{pmatrix} = E_1 = \operatorname{span}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

d. *3 points.* Determine whether A is diagonalizable. If A is not diagonalizable, explain why not. If A is diagonalizable, find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$ and $A = PDP^{-1}$.

A is diagonalizable since $\dim E_{-2} + \dim E_1 = 3$ and A is 3×3 matrix

$AP = PD$ Check

$$\begin{pmatrix} -5 & -6 & 3 \\ 3 & 4 & -3 \\ 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} \cancel{-2} & & \\ & \cancel{-2} & \\ - & & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 1 \\ 0 & -2 & -1 \\ -2 & -4 & 0 \end{pmatrix}$$

$$\cancel{\begin{pmatrix} -2 & 0 & 1 \\ 0 & -2 & -1 \\ -2 & -4 & 0 \end{pmatrix}} = \begin{pmatrix} -2 & 0 & 1 \\ 0 & -2 & -1 \\ -2 & -4 & 0 \end{pmatrix}$$

same

Finding P^{-1} sucks, but you don't need to do it!