Math 214 Spring 2007 **SHOW ALL YOUR WORK** **BONUS** Quiz 6

1. **TRUE or FALSE** – put your answer in the box. That answer is worth 1 point. To receive ANY credit, you must also give a brief, and correct, explanation in support of your answer! Remember a statement is TRUE only if it is ALWAYS true, and it is FALSE if there exists an example which makes it FALSE.

(a) (*3 points*) For any $n \times n$ matrix A there exists a real number λ and a $n \times 1$ vector $\vec{x}$ such that $A\vec{x} = \lambda\vec{x}$.

FALSE/TRUE

A real matrix can have eigenvalues that are not real numbers.

FALSE

counter example: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $p(\lambda) = \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1 = 0$

$\lambda = \pm i$

TRUE If $\vec{x} = \vec{0}$ then $A\vec{0} = \lambda\vec{0}$ for all $n \times n$ matrices A and all real numbers λ. So, technically the statement IS true if one includes the zero vector as a possibility.

(b) (*3 points*) A is singular (not invertible) if and only if A has at least one zero eigenvalue. In other words, IF A is singular, THEN A has at least one zero eigenvalue *AND* IF A has at least one zero eigenvalue, THEN A is singular.

TRUE

A^{-1} exists $\Leftrightarrow \det A \neq 0 \Leftrightarrow \prod_{i=1}^{m} \lambda_i \neq 0 \Leftrightarrow \forall \lambda_i \neq$

A^{-1} exists $\Leftrightarrow$ Every $\lambda_i \neq 0$

A^{-1} does not exist $\Leftrightarrow$ There exist a $\lambda_i = 0$

Or you can prove each direction separately

A singular $\Rightarrow \det(A) = 0 \Rightarrow \det(A - 0I) = 0 \Rightarrow p(0) = 0 \Rightarrow 0$ is an eigenvalue of A

$\lambda = 0 \Rightarrow \prod_{i=1}^{n} \lambda_i = 0 \Rightarrow \prod \lambda_i = \det(A) = 0 \Rightarrow A$ is singular

(c) (*4 points*) The eigenvectors of A^T are the same as the eigenvectors of A.

FALSE

The eigenvalues of A^T and A are the same but the eigenvectors are different (unless $A = A^T$).

counter example

$A = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$ $\lambda(A) = 1, 2$

$A^T = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$ $\lambda(A) = 1, 2$

$\text{null}(A - 1I) = \text{null}\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} = \text{span}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = E_1(A)$

$\text{null}(A - 2I) = \text{null}\begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} = \text{span}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = E_2(A)$

$\text{null}(A^T - 1I) = \text{null}\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} = \text{span}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = E_1(A^T)$

$\text{null}(A^T - 2I) = \text{null}\begin{pmatrix} -1 & 0 \\ \end{pmatrix} = \text{span}\begin{pmatrix} 0 \\ \end{pmatrix} = E_2(A^T)$