Math 214 Spring 2007 **SHOW ALL YOUR WORK** **BONUS** Quiz 2

1. *6 points.* Consider the linear system

$$\begin{aligned} 4x - 2y + z &= a \\ x + y + z &= b \end{aligned}$$

where a and b are real numbers. Our goal is to discover a relationship between the solution sets of this system for various values of a and b.

a. *2 points.* Consider the case $a = b = 0$. This is known as the **homogeneous** case. Use Gaussian Elimination to solve the system.

$$\left(\begin{array}{ccc|c} 4 & -2 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 4 & -2 & 1 & 0 \end{array}\right) \xrightarrow{R_2' = R_2 - 4R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -6 & -3 & 0 \end{array}\right) \xrightarrow{R_2' = R_2/-3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right)$$

$$\xrightarrow{R_1' = R_1 - \frac{1}{2}R_2} \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 2 & 1 & 0 \end{array}\right) \quad \begin{aligned} x + \tfrac{1}{2}z &= 0 \\ 2y + z &= 0 \end{aligned} \quad \begin{aligned} x &= -\tfrac{1}{2}z \\ y &= -\tfrac{1}{2}z \end{aligned} \quad z = t \quad \vec{x} = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix} t = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} t$$

b. *2 points.* What is the geometric interpretation or "shape" of the solution? Is it a point in $\mathbb{R}^2$? A point in $\mathbb{R}^3$? A line in $\mathbb{R}^2$? A line in $\mathbb{R}^3$? A plane in $\mathbb{R}^3$? Something else?

The solution is a line through the origin in $\mathbb{R}^3$

c. *2 points.* Express your solution in vector form, i.e. $\vec{x} = \vec{p} + t\vec{d}$.

$$\vec{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$$

2. *4 points.* Choose any non-zero value of a and b that you like. This is known as the **non-homogeneous** case.

a. *2 points.* Repeat **Question 1** (i.e. Use Gaussian Elimination to solve the system with your chosen values of a and b) and express your answers in vector form, i.e. $\vec{x} = \vec{p} + t\vec{d}$.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 4 & -2 & 1 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -6 & -3 & -3 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} \end{array}\right)$$

$$\begin{aligned} x + \tfrac{1}{2}z &= \tfrac{1}{2} \\ y + \tfrac{1}{2}z &= \tfrac{1}{2} \end{aligned} \quad \begin{aligned} x &= \tfrac{1}{2} - \tfrac{1}{2}z \\ y &= \tfrac{1}{2} - \tfrac{1}{2}z \end{aligned} \quad z \text{ free} \quad \vec{x} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} + t \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix}$$

A line in $\mathbb{R}^3$ NOT thru origin

b. *2 points.* What is the (geometric) relationship between your solutions in **1(c)** and **2(a)**? In other words, how are the solutions to the homogeneous linear system and non-homogeneous linear system related? EXPLAIN YOUR ANSWER.

The solution to the non-homogeneous system is a line parallel (same direction vector) but shifted so that it does not go through the origin, as the solution to the homogeneous system.