
Linear Systems

Math 214 Spring 2006
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Fowler 307 MWF 2:30pm - 3:25pm
<http://faculty.oxy.edu/ron/math/214/06/>

Class 33: Friday April 28

TITLE Wrapping it all together!

CURRENT READING Poole

Summary

The text uses the repetitive theme of adding statements to the Fundamental Theorem of Invertible Matrices as a theme. We'll look at the **final** version.

Theorem 7.19

The Fundamental Theorem of Invertible Matrices (Final Version). Let A be a $n \times n$ matrix. Each of the following statements is equivalent:

- (a) A is invertible.
- (b) $A\vec{x} = \vec{b}$ has a unique solution for every \vec{b} in \mathbb{R}^n .
- (c) $A\vec{x} = \vec{0}$ has only the trivial solution.
- (d) The reduced row echelon form of A , $\text{rref}(A)$, is I_n .
- (e) A is a product of elementary matrices.
- (f) $\text{rank}(A) = n$.
- (g) $\text{nullity}(A) = 0$.
- (h) The column vectors of A are linearly independent.
- (i) The column vectors of A span \mathbb{R}^n .
- (j) The column vectors of A form a basis for \mathbb{R}^n .
- (k) The row vectors of A are linearly independent.
- (l) The row vectors of A span \mathbb{R}^n .
- (m) The row vectors of A form a basis for \mathbb{R}^n .
- (n) The determinant of A is not equal to zero.
- (o) 0 is not an eigenvalue of A .
- ⋮
- (u) 0 is not a singular value of A .

Note that the book's version has some more concepts dealing with Linear Transformations and Changes of Basis that we did not discuss in this version of the course.