Linear Systems

Math 214 Spring 2006 — Fowler 307 MWF 2:30pm - 3:25pm
 — http://faculty.oxy.edu/ron/math/214/06/

Class 1: **Monday January 23**

SUMMARY Scalars and Vectors
CURRENT READING Poole 1.1 and 1.2

INTRO
In today's class we review the concepts of vectors and scalars. In addition, we introduce the central idea of a **linear combination** of vectors.

Homework Assignment #1
Section 1.1 # 1d, 2d, 3c, 4c, 5a, 6, 9, **11, 15, 17, 20** *: DUE WED JAN 25*
EXTRA CREDIT #14

What is a vector?

Roughly speaking, a vector is just a "bunch of numbers"!
More precisely, a vector is an *ordered set of numbers.*

Example 1. $[2\ \ 0]$ is a vector; $[0\ \ 2]$ is also; these are two different vectors, since order matters.

$[2\ \ -5\ \ 7.1]$ is a **row vector**; $\begin{bmatrix} 2 \\ -5 \\ 7.1 \end{bmatrix}$ is a **column vector**.

Q: What's the difference between a row vector and a column vector?

Note. To save space, we sometimes write $(4, 0, -8)$ instead of $\begin{bmatrix} 4 \\ 0 \\ -8 \end{bmatrix}$. So $(4, 0, -8)$ is a column vector.

Each number in the vector is called a **component** of the vector.
Q: what's the second component of the vector $[3\ \ 6\ \ 0]$? **Ans:**

Vectors are used to represent many different things!

Example 2. Start from home. Drive 6 miles East, 2 miles North. Represent this by the vector $[6\ \ 2]$. Then continue driving 3 miles East, 5 miles South. Represent this by $[3\ \ -5]$.
Q: Where are we relative to home? **Ans:** Add the two vectors: $[6\ \ 2] + [3\ \ -5] = [9\ \ -3]$.

(Draw picture)

• We add vectors component-wise: one component at a time.

Example 3. I have 4 nickels, 3 dimes, and 2 quarters. You give me 3 nickels and 1 dime, and take 1 quarter. So I'm left with: $[4\ \ 3\ \ 2] + [3\ \ 1\ \ -1] = [7\ \ 4\ \ 1]$.

Note. The book uses boldface letters for vectors. It is difficult to *write* in boldface. So instead we'll use "arrow notation" for vectors:
Book: Let $\mathbf{v} = [4\ \ 3]$. Let $\mathbf{w} = [5\ \ 3]$. Then $\mathbf{v} + \mathbf{w} =$?
Us: Let $\vec{v} = [4\ \ 3]$. $\vec{w} = [5\ \ 3]$. Then $\vec{v} + \vec{w} =$?

Example 4. $[4\ \ 2] + [3\ \ 1\ \ -1] =$? **Ans:** Undefined.

Vectors of different size can NOT be added to each other.

Multiplying a vector by a number: scalars

What's $5 + 5 + 5 + 5 + 5 + 5 =$?
What's $[5\ \ 3] + [5\ \ 3] + [5\ \ 3] + [5\ \ 3] + [5\ \ 3] + [5\ \ 3] =$?
So, what's $6[5\ \ 3] =$? **Ans:**

Here the number 6 is called a **scalar**. Why? Because if you draw both vectors, $[5\ \ 3]$ and $[30\ \ 18]$, on two separate xy-planes, they'll have different lengths but the same direction (slope): we're only changing the "scale on our map" to make one vector look like the other.

Subtracting vectors

Example 5. Let $\vec{v} = [4\ \ 3]$. $\vec{w} = [5\ \ 3]$. Then $\vec{v} - \vec{w} =$? Ans: $[-1\ \ 0]$.

How can we represent vector subtraction pictorially?

Step 1. Draw $\vec{v}$.
Step 2. Multiply $\vec{w}$ by -1.
Step 3. Add $-\vec{w}$ to $\vec{v}$.

Linear Combinations

Example 6. Find a and b such that $a[5\ \ 3] + b[3\ \ 2] = [0\ \ 1]$.

Ans: Solve two equations with two unknowns:
$5a + 3b = 0$
$3a + 2b = 1$.

We get: $a = -3$, $b = 5$.

So $(-3)[5\ \ 3] + (5)[3\ \ 2] = [0\ \ 1]$. We say $[0\ \ 1]$ is a *linear combination* of $[5\ \ 3]$ and $[3\ \ 2]$.
(Books sometimes just say combination, instead of linear combination.)

Definition 1. Let $\vec{v_1}, \cdots, \vec{v_n}$ be vectors. To say a vector $\vec{w}$ is a **linear combination** of $\vec{v_1}, \cdots, \vec{v_n}$ means there exist scalars $c_1, \cdots, c_n \in \mathbb{R}$ such that $c_1\vec{v}_1 + \cdots c_n\vec{v_n} = \vec{w}$. The numbers $c_1, \cdots, c_n$ are called **coefficients**.

Example 7. Is $[5\ \ 6\ \ 0]$ a linear combination of $[1\ \ 0\ \ 0]$, $[0\ \ 3\ \ 0]$, and $[0\ \ 0\ \ 8]$? **Ans:**

Example 8. Is $[5\ \ 6\ \ 0]$ a linear combination of $[0\ \ 1\ \ 1]$, $[0\ \ 3\ \ 0]$, and $[0\ \ 0\ \ 8]$? **Ans:**

Example 9. What are all possible lin combs of $[1\ \ 0]$ and $[0\ \ 1]$? **Ans:**

Example 10. What are all possible lin combs of $[1\ \ 1]$ and $[2\ \ 2]$? **Ans:**
