1. **(a)** Show that the plane given by $4x - y - z = 6$ and the line given by $x = t, y = 1 + 2t, z = 2 + 3t$ intersect.

Substitute $x = t$, $y = 1+2t$, $z = 2+3t$ into $4x - y - z = 6$

$$4(t) + (1+2t) - (2+3t) = 6$$
$$4t - 1 - 2t - 2 - 3t = 6$$
$$-3 - t = 6$$
$$-t = 9$$
$$t = -9$$

When $t = 9$, $x = -9$
$$y = 1 + 2(-9) = -17$$
$$z = 2 + 3(-9) = -25$$

$\begin{pmatrix} -9 \\ -17 \\ -25 \end{pmatrix}$ is on the line

$$4(-9) - (-17) - (-25) \overset{?}{=} 6$$
$$-36 + 17 + 25 \overset{?}{=} 6$$
$$-36 + 42 \overset{?}{=}$$
$$6 \overset{?}{=} 6 \checkmark$$

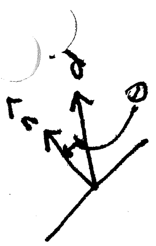
$\begin{pmatrix} -9 \\ -17 \\ -25 \end{pmatrix}$ is also on the plane

(b) Find the acute angle of intersection between the line and the plane given in part (a).

Given $4x - y - z = 6$, the vector $\begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix}$ is orthogonal (or normal) to the plane

The line $\begin{pmatrix} t \\ 1+2t \\ 2+3t \end{pmatrix} = t\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$, thus $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is the direction the line is in.

The angle between the line and plane is given by the dot product formula



$$\cos\theta = \frac{(4,-1,-1)\cdot(1,2,3)}{|(4,-1,-1)||(1,2,3)|} = \frac{4-2-3}{\sqrt{4^2+1^2+1^2}\sqrt{1^2+2^2+3^2}} = \frac{-1}{\sqrt{18}\sqrt{14}}$$

$= \frac{-1}{[illegible]} = \frac{-1}{6\sqrt{7}}$, $\theta = 93.6°$ [illegible] 1.63 rad Actual Angle is $|90° - \theta| = 3.6°$