Report on Test 1

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Point Distribution (N=22)

Range	100+	92+	89+	87+	82+	80+	77+	73+	70+	65+	60+	55+	55-
Grade	A+	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Frequency	0	1	6	2	2	2	1	2	0	2	2	1	1

Summary Overall class performance was pretty good. The mean score was 77. The high score was a 94.

#1 Span, Linear Independence, Rank. This is a short answer question. It is asking about the implications of rref$(A) = \mathcal{I}$. **(a)** The rank of A is the number of non-zero rows in rref(A), which in this case is 3. **(b)** The span of the columns of A is the set of all possible linear combinations of the columns. Since rref$(A) = \mathcal{I}$, the three columns of A are linearly independent and every vector in $\mathbb{R}^3$ can be written as a linear combination of these column. Thus the span of the columns is equal to $\mathbb{R}^3$. **(c)** Since the span of the columns is $\mathbb{R}^3$ the columns are linearly independent. **(d)** Since rref$(A) = \mathcal{I}$ using Gauss-Jordan on the augmented matrix $A|I$ will produce $I|A^{-1}$.

#2 Dot Product, Magnitude, Lengths. This question is about thinking about the definition of the magnitude of vectors and the definition of the dot product. Basically, what is the significance of the $\cos(\theta)$ term in the $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos(\theta)$ formula? **(a)** Commutativity. $\vec{u} \circ \vec{v} = \vec{v} \circ \vec{u}$ This is true for both the Buckmire product and the Dot product since in multiplication between real numbers order is irrelevant. **(b)** $(c\vec{u}) \circ \vec{v} = c\vec{u} \circ \vec{v}$ Distributivity of Scalar Multiplication. In this case $(c\vec{u}) \circ \vec{v} = |c\vec{u}||\vec{v}| = |c||\vec{u}||\vec{v}|$ which is only equal to $c\vec{u} \circ \vec{v}$ if $|c| = c$ which would only occur if c is greater than zero. **(c)** Distributity of Vector Addition. $\vec{u} \circ (\vec{v} + \vec{w}) = \vec{u} \circ \vec{v} + \vec{u} \circ \vec{w}$ This is not true for Buckmire product but is true for the Dot product. That's because $\vec{u} \circ (\vec{v} + \vec{w}) = |\vec{u}||\vec{v} + \vec{w}|$ but $|\vec{v} + \vec{w}| \neq |\vec{v}| + |\vec{w}|$. **(d)** $\vec{u} \circ \vec{u} \geq 0$ is true for the Buckmire product and the Dot product since the multiplication of any two lengths must be greater than or equal to zero. **(e)** $\vec{u} \circ \vec{u} = 0 \Leftrightarrow \vec{u} = \vec{0}$ is true for both the Buckmire product and the Dot product since the multiplication of the length of a vector with itself can only be zero if the vector itself is zero.

#3 Matrix Operations. The main difference between arithmetic operations using matrices to arithmetic operations using numbers is that multiplication is not *a priori* defined for every $m \times n$ matrix. The identity matrix must be square but the zero matrix does NOT have to be. Recall this is a TRUE or FALSE question. For FALSE, all you have to do is find one counter example. For TRUE, you have to prove the given statement is always TRUE. **(a)** "If $A^2 = \mathcal{O}$ then $A = \mathcal{O}$." **FALSE.** This zero matrix must be square since it is a result of a matrix squared. There does exist atleast one matrix $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ which when multiplied by itself will equal $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. **(b)** "If $A = \mathcal{O}$ then $A^2 = \mathcal{O}$." **FALSE.** If $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ then A^2 is not defined, so A^2 is not equal to $\mathcal{O}$ for every matrix A **(c)** "If $A^2 = \mathcal{I}$ then $A = \mathcal{I}$." **FALSE.** Lots of students recognized that this implies that $A^{-1} = A$. (Does A^{-1} exist for every matrix? No!) It is true that the identity matrix has this property, but there are lots of other matrices which have this property also, for example $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. **(d)** "If $A = \mathcal{I}$ then $A^2 = \mathcal{I}$." **TRUE.** If $A = \mathcal{I}$ then $A^2 = \mathcal{I}^2 = \mathcal{I}$

#4 Parametric equations, lines, planes, subspaces. Given $x = t + 1, y = 2t - 3, z = 3t$ then $x - 1 = t$ and $y + 3 = 2t$ and $z = 3t$ so that $2(x - 1) = y + 3$ $3(y + 3) = 2z$ and $z = 3(x - 1)$. These three linear equations in three variables represent the intersection of three planes along a line. **(b)** The intersection is a 1-dimensional object, since it is described using one unknown parameter (t in this case). The geometric interpretation is that this is a line. The vector equation is $\vec{x} = t\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$. **(c)** A subspace of $\mathbb{R}^3$ must contain the zero vector $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. This line does not go through the origin, so it is not a subspace of $\mathbb{R}^3$. (If you check the other conditions for a subspace: closure under scalar multiplication and closure under vector addition it fails to be a subspace under those definitions also.)