

Worksheet 19

TITLE Iterated Integration
CURRENT READING McCallum, Section 16.2
HW #8 (DUE Wednesday 04/01/15 5PM)
McCallum, *Section 16.1*: 2, 4, 6, 7, 8, 14, 22, 23..
McCallum, *Chapter 16.2*: 1, 3, 4, 7, 11, 13, 14, 16, 18, 19, 23, 33, 34, 37, 38, 43, 50.

SUMMARY
This worksheet discusses how to evaluate multiple integrals, which are often called "iterated integrals" because evaluation involves repeating (or iterating) the integration process, using each variable and its associated limits. The iterated integration process can be related conceptually to the inverse of partial differentiation.

THEOREM

Fubini's Theorem for Double Integrals

The double integral of a continuous function $f(x, y)$ over a rectangle $\mathcal{R} = [a, b] \times [c, d]$ is equal to the iterated integral (computed in either order).

$$I = \int_a^b \int_c^d f(x,y)dy\,dx = \int_a^b \left[\int_c^d f(x,y)dy\right] dx \quad = \quad \int_a^b F(x)\,dx$$
$$= \int_c^d \int_a^b f(x,y)dx\,dy = \int_c^d \left[\int_a^b f(x,y)dx\right] dy \quad = \quad \int_a^b G(y)\,dy = I$$

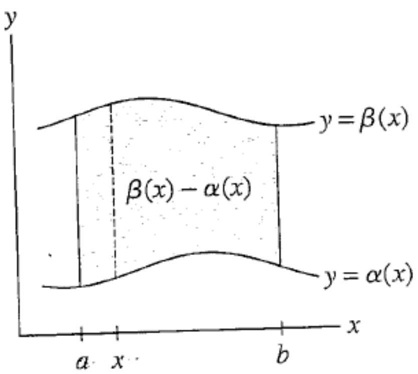
EXAMPLE

Evaluate $\int_0^1 \int_1^2 x^2 + y\,dx\,dy$ two different ways to illustrate the result above.

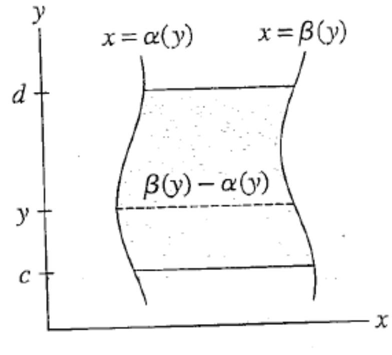
Exercise

Evaluate $\int_0^2 \int_0^1 ye^{xy}\,dx\,dy$

What happens if the region of interest of the iterated integral is non-rectangular?



(A) Vertically simple region
$a \le x \le b,\ \alpha(x) \le y \le \beta(x)$



(B) Horizontally simple region
$c \le y \le d,\ \alpha(y) \le x \le \beta(y)$

Iterated Integration Over Non-Rectangular Regions

To integrate $f(x, y)$ over a "y-simple" (or vertically simple) region defined as $a \le x \le b \cap u(x) \le y \le v(x)$ use $\displaystyle\int_a^b \int_{u(x)}^{v(x)} f(x, y)dydx$

To integrate $f(x, y)$ over a "x-simple" (or horizontally simple) region defined as $r(y) \le x \le s(y) \cap c \le y \le d$ use $\displaystyle\int_c^b \int_{r(y)}^{s(y)} f(x, y)dxdy$

EXAMPLE

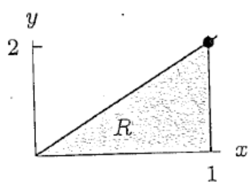
Find an expression (or two!) for the mass of a triangular metal plate whose density $\delta(x, y)$ varies with respect to x and y. The triangular plate has vertices at ((0,0), (0,2) and (1,0).

`QUESTION:` Is the triangular area of integration for δ an x-simple or y-simple region?

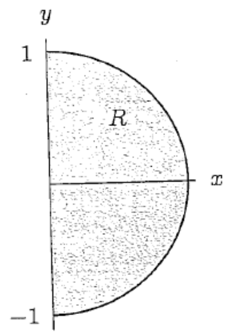
GROUPWORK

Adapted from **McCallum, page 882, Exercise 28-31.** Integrate the function $f(x, y) = xy$ over the following areas given in 28-31.**Describe whether these areas are x-simple, y-simple, both or neither.**

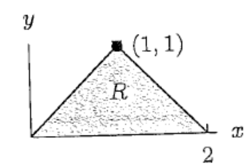
28.



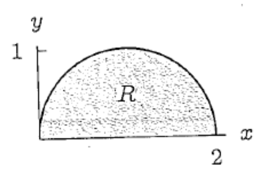
29.



30.



31.



GROUPWORK

McCallum, page 881, Exercise 13-16. Sketch the region of integration and evaluate the integral.

13. $\int_1^3 \int_0^4 e^{x+y}\, dy\, dx$

14. $\int_0^2 \int_0^x e^{x^2}\, dy\, dx$

15. $\int_1^5 \int_x^{2x} \sin x\, dy\, dx$

16. $\int_1^4 \int_{\sqrt{y}}^{y} x^2 y^3\, dx\, dy$