Multivariable Calculus

Class 4

TITLE Functions, Vector Functions, Scalar Functions and $f(x, y)$ as surfaces in $\mathbb{R}^3$
CURRENT READING McCallum, Section 12.1 to 12.2
HW #2 (DUE WED 02/04/15)
McCallum, Section 13.3: 2, 5, 6, 10, 20, 22, 29, 35, 38, 81*.
McCallum, Section 13.4: 3, 4, 13, 15, 18, 20, 32, 51, 64*.
McCallum, Section 17.1: 7, 10, 13, 16, 36, 50*.
SUMMARY In today's class we will begin to learn about functions of two variables, using our intuition from single-variable Calculus to interpret graphs of functions of two variables as surfaces.

DEFINITION: **function**

A **function** consists of a pre-image or **domain** (the set of input values), a range or **image** (the set of output values) and a **rule** assigning a unique output value to to each input value.

Exercise

Write down an example of a function. Explicitly state what the domain, image and rule are for your choice.

Vector Functions of a Scalar Variable

A **vector** function f of a **scalar** variable $\vec{f}(x)$ with **domain** $D \subset \mathbb{R}$ and **image** $R \subset \mathbb{R}^n$ means that the function f has possible input values which form a subset of the real numbers and the set of possible output values are a subset of $\mathbb{R}^n$, i.e. vectors. Often the notation $f : D \to R$ or $f : \mathbb{R} \to \mathbb{R}^n$ is used.

EXAMPLE

What kind of geometric object is the image of the function $\vec{x}(t) = (1 + 3t, -1 - t, -2 + t)$?

NOTE If the functions in the components of the vector function $\vec{x}(t)$ are not linear functions of the variable t (often called the **parameter**), then this 1-dimensional geometric object is called **a parametric curve** in $\mathbb{R}^n$)

Scalar Functions of a Vector Variable

A **scalar** function f of a **vector** variable $f(\vec{x})$ with **domain** $D \subset \mathbb{R}^n$ and **image** $R \subset \mathbb{R}$ means that it has possible input values that are vectors in $\mathbb{R}^n$ and that the set of possible output values are real numbers. Often the notation $f : D \to R$ or $f : \mathbb{R}^n \to \mathbb{R}$ is used.

DEFINITION: **graph**

The **graph** of a scalar function of a vector variable $f(\vec{x})$ is defined to be the set of ordered pairs $(\vec{x}, f(\vec{x}))$ where $\vec{x}$ is in the domain of f. In this case we say that the graph of f is **explicitly** represented by f. A graph is a **visual representation** of a function.

QUESTION: What are some other ways to represent a function in addition to a graph?

QUESTION: Can a function be treated as an object? If so, give an example of this practice!

In practice the only scalar functions of a vector variable that we can really get a good handle on visually are either of the type $f : \mathbb{R} \to \mathbb{R}$ or $f : \mathbb{R}^2 \to \mathbb{R}$. These graphs are represented by ordered pairs that look like $(x, f(x))$ and $(x, y, f(x, y))$ respectively.

DEFINITION: **surface**

We know all about the first case ($f : \mathbb{R} \to \mathbb{R}$)from single-variable Calculus so we will be concentrating on the second case ($f : \mathbb{R}^2 \to \mathbb{R}$), which are often called **surfaces** and denoted $z = f(x, y)$ so that the ordered pair looks like (x, y, z). Below are two examples of surfaces in $\mathbb{R}^3$.