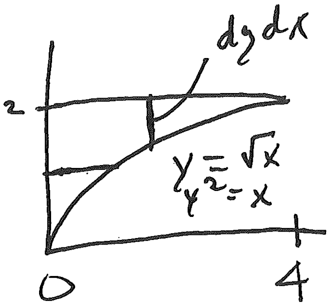
Math 212 Spring 2015 **SHOW ALL YOUR WORK** Quiz NINE

**1.** (*5 points*) Evaluate $\int_0^4 \int_{\sqrt{x}}^2 \sin(y^3)\, dy\, dx$. (**HINT: there does not exist any explicit function $F(y)$ whose derivative $F'(y)$ equals $\sin(y^3)$ but this integral is calculable after applying Fubini's Theorem.**)




$$\int_0^4\int_{\sqrt{x}}^2 \sin(y^3)\,dy\,dx$$

$$= \int_0^2\int_0^{y^2} \sin(y^3)\,dx\,dy$$

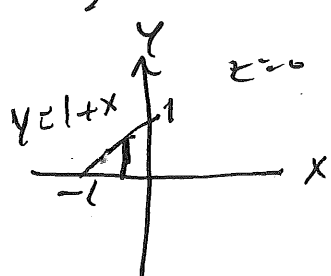
$$= \int_0^2 \sin(y^3)\,x\Big|_0^{y^2} dy = \int_0^2 \sin(y^3)\,y^2\,dy$$

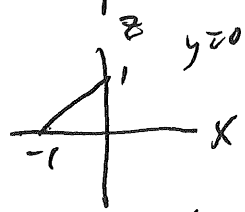
$$= \frac{1}{3}\int_0^2 3y^2\sin(y^3)\,dy = \frac{1}{3}\,-\cos(y^3)\Big|_0^2$$

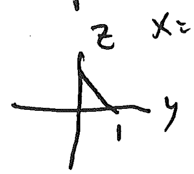
$$= -\frac{1}{3}\cos(8) + \frac{1}{3}$$

**2.** (*5 points*) Show that the volume of the tetrahedron bounded by the planes $y = 0$, $z = 0$, $x = 0$ and $y - x + z = 1$ is $1/6$ by writing down and evaluating an appropriate iterated integral. (**HINT: Draw a picture indicating where the tetrahedron crosses the $x$, $y$ and $z$-axes.**)

$y - x + z = 1$




$$\int_{-1}^0\int_0^{1+x}\int_0^{1+x-y} dz\,dy\,dx = \int_{-1}^0\int_0^{1+x} 1+x-y\;dy\,dx$$

$$= \int_0^1\int_{y-1}^0 1+x-y\;dx\,dy$$

$$= \int_0^1 x+\frac{x^2}{2}-xy\Big|_{y-1}^0 dy$$

$$= \int_0^1 1-y-\frac{(y-1)^2}{2}+y(y-1)\,dy$$

$$= \int_0^1 1-y-\frac{y^2}{2}+y-\frac{1}{2}+y^2-y\,dy$$

$$= \int_0^1 \frac{1}{2}-y+\frac{y^2}{2}\,dy$$

$$= \frac{y}{2}-\frac{y^2}{2}+\frac{y^3}{6}\Big|_0^1 =$$

$$= \frac{1}{2}-\frac{1}{2}+\frac{1}{6} = \frac{1}{6}$$

$$= \int_{-1}^0 y+xy-\frac{y^2}{2}\Big|_{y=0}^{y=1+x} dx$$

$$= \int_{-1}^0 1+x+x(1+x)-\frac{(1+x)^2}{2}\,dx$$

$u = 1+x$ $du = dx$

$$\int_0^1 u+u(u-1)-\frac{u^2}{2}\,du$$

$$= \int_0^1 \frac{u^2}{2}\,du$$

$$= \frac{u^3}{6}\Big|_0^1 = \frac{1}{6}$$