Math 212 Spring 2015 **SHOW ALL YOUR WORK** Quiz **SEVEN**

1. Consider the function $S(x, y) = 3xe^y - x^3 - e^{3y}$.

(a) (*2 points*) Compute $\vec{\nabla} S$.

$$\vec{\nabla} S = S_x \hat{i} + S_y \hat{j}$$
$$= (3e^y - 3x^2)\hat{i} + (3xe^y - 3e^{3y})\hat{j}$$

(b) (*2 points*) Use your answer from **(a)** to show that there is only one critical point of $S(x, y)$, at the point $(1, 0, 1)$.

$S_x = 3e^y - 3x^2 = 0 \Rightarrow e^y = x^2$

$S_y = 3xe^y - 3e^{3y} = 0 \Rightarrow xe^y = e^{3y} \Rightarrow x \cdot x^2 = (x^2)^3$

$x^3 = x^6$

Critical Point is

$x = 1, y = 0, z = S(1,0) = 3 \cdot 1 \cdot e^0 - 1^3 - 1$

$= 3 - 1 - 1$

$z = 1$

$x = 1$ or $x = 0$

when $x = 1$, $e^y = 1 \Rightarrow y = 0$

when $x = 0$, $e^y = 0$ no sol^n for y

(c) (*4 points*) Use the second derivative test $D = f_{xx}(a, b) f_{yy}(a, b) - f_{xy}^2(a, b)$ to classify the critical point you found in part **(b)** as a **local maximum** of $S(x, y)$.

$S_{xx} = -6x$

$S_{yy} = 3xe^y - 9e^{3y}$

$S_{xy} = 3e^y$

At $x = 1, y = 0$

$S_{xx} = -6$

$S_{yy} = 3 \cdot 1 e^0 - 9 \cdot e^0 = -6$

$S_{xy} = 3 \cdot e^0 = 3$

$D = -6 \cdot -6 - 3^2$

$= 36 - 9 = 27 > 0 \Rightarrow$ MAX or MIN

$S_{xx} < 0$ and $S_{yy} < 0$

This is a local ~~maximum~~ maximum

(d) (*2 points*) Is the local maximum you found in part **(c)** also a global maximum of $S(x, y)$? EXPLAIN YOUR ANSWER AND THEN COMPARE AND CONTRAST THIS SITUATION INVOLVING A MULTIVARIABLE FUNCTION WITH ONE CRITICAL POINT THAT IS A LOCAL MAXIMUM WITH A SIMILAR SITUATION OF A SINGLE VARIABLE FUNCTION WITH A SINGLE CRITICAL POINT THAT IS A LOCAL MAXIMUM.

No, it's NOT a global max.

$S(-3,0)$ is ~~less~~ greater than $S(1,0) = 1$.

$= -9 + 27 - 1 = 17 > 1$

For single variable functions, if you have one critical point that local max will be a global max also. However, the same is not true for multivariable functions.