Math 212 Spring 2015 **SHOW ALL YOUR WORK** Quiz **FIVE**

1. Suppose that in a certain region of space the electric potential V is given by

$$V(x, y, z) = 5x^2 - 3xy + xyz.$$

(a) (*3 points*) Compute $\vec{\nabla} V$, i.e. the gradient vector of the electric potential V.

$$\vec{\nabla} V = V_x \hat{\imath} + V_y \hat{\jmath} + V_z \hat{k}$$

$$= \begin{pmatrix} 10x - 3y + yz \\ -3x + \cancel{x} z \\ xy \end{pmatrix}$$

(b) (*3 points*) Find $V_{\vec{u}}(1, 2, 3)$, the rate of change of the electric potential V at the point $P(1, 2, 3)$ in the direction of the vector $\vec{u} = \hat{\imath} + \hat{\jmath} - \hat{k}$.

$$\vec{\nabla} V(1,2,3) = \begin{pmatrix} 10 \cdot 1 - 3 \cdot 2 + 2 \cdot 3 \\ -3 \cdot 1 + 1 \cdot 3 \\ 1 \cdot 2 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \\ 2 \end{pmatrix} = 10\hat{\imath} + 2\hat{k}$$

$$\|\vec{u}\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$V_{\vec{u}} = \vec{\nabla} V(1,2,3) \cdot \hat{u} = \begin{pmatrix} 10 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \cdot \frac{1}{\sqrt{3}}$$

$$\hat{u} = \frac{\vec{u}}{\|\vec{u}\|}$$

$$= \frac{1}{\sqrt{3}}(10 + 0 - 2) = \frac{8}{\sqrt{3}}$$

(c) (*2 points*) In which direction does the electric potential V change the most rapidly at the point $P(1, 2, 3)$?

The gradient vector points in direction of maximum change.

$$\vec{\nabla} V(1,2,3) = 10\hat{\imath} + 2\hat{k}$$

(d) (*2 points*) What is the maximum rate of change of the electric potential V at the point $P(1, 2, 3)$?

The maximum rate of change is the magnitude of the gradient vector,

$$\|\vec{\nabla} V(1,2,3)\| = \|10\hat{\imath} + 2\hat{k}\|$$

$$= \sqrt{10^2 + 2^2}$$

$$= \sqrt{104} = 2\sqrt{26}$$