Math 212 Fall 2014 **SHOW ALL YOUR WORK** **BONUS** Quiz THREE

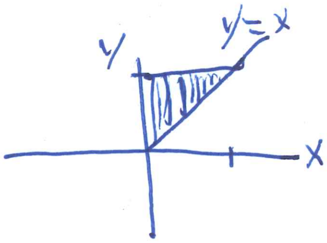
1(a) (*1 point*) Show that the integral $\int_0^1 \int_x^1 \int_0^y dz\, dy\, dx = \frac{1}{3}$.

$$\int_0^1 \int_x^1 y\, dy\, dx = \int_0^1 \frac{y^2}{2}\Big|_x^1 dx = \int_0^1 \frac{1}{2} - \frac{x^2}{2} dx$$

$$= \frac{1}{2}x - \frac{x^3}{6}\Big|_0^1 = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$$

1(b) (*2 points*) Change the order of integration of the integral in **(a)** and evaluate this new integral to confirm Fubini's Theorem.

$$\int_0^1 \int_x^1 y\, dy\, dx = \int_0^1 \int_0^y y\, dx\, dy = \int_0^1 xy\Big|_0^y dy$$

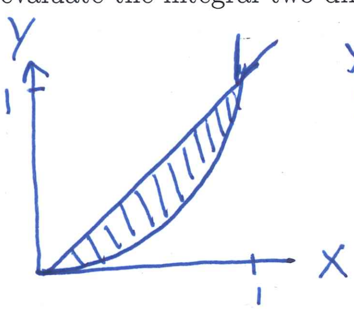


$$= \int_0^1 y^2 dy$$

$$= \frac{y^3}{3}\Big|_0^1$$

$$= \frac{1}{3}$$

2. (*2 points*) Consider the integral $\int_0^1 \int_y^{\sqrt{y}} 2xy\, dx\, dy$. Sketch the region being integrated and evaluate the integral two different ways.



$x = \sqrt{y}$
$x^2 = y$
$x = y$

$$\int_0^1 x^2 y\Big|_y^{\sqrt{y}} dy = \int_0^1 y(y - y^2)\, dy$$

$$= \int_0^1 y^2 - y^3\, dy = \frac{y^3}{3} - \frac{y^4}{4}\Big|_0^1$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$$

$$\int_0^1 \int_{x^2}^{x} 2yx\, dy\, dx = \int_0^1 y^2 x\Big|_{x^2}^{x} dx$$

$$= \int_0^1 x(x^2 - x^4)\, dx = \int_0^1 x^3 - x^5 dx$$

$$= \frac{1}{4}x^4 - \frac{1}{6}x^6\Big|_0^1 = \frac{1}{4} - \frac{1}{6} = \frac{2}{24}$$

$$= \frac{1}{12}$$