Math 212 Fall 2014 **SHOW ALL YOUR WORK** **BONUS** Quiz ONE

Adapted from Question 1 on Exam 1 for Math 212 Fall 2014.

a. (*1 point*) Show that the equation of the plane $\mathcal{P}$ ~~that~~ containing the points $A(1,0,0)$, $B(1,2,-2)$, and $C(0,-3,4)$ is $x+y+z=1$. **HINT: You do NOT Have to compute a vector cross product to answer this question!**

$1+0+0=1=1$ ✓

$1+2-2=1=1$ ✓

$0+-3+4=1=1$ ✓

$\vec{B}-\vec{A}=\begin{pmatrix}1\\2\\-2\end{pmatrix}-\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}0\\2\\-2\end{pmatrix}$

$\vec{C}-\vec{A}=\begin{pmatrix}0\\-3\\4\end{pmatrix}-\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}-1\\-3\\4\end{pmatrix}$

$\vec{n}\cdot(\vec{C}-\vec{A})=\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\-3\\4\end{pmatrix}=0$ ✓

$\vec{n}\cdot(\vec{B}-\vec{A})=\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}0\\2\\-2\end{pmatrix}=0$ ✓

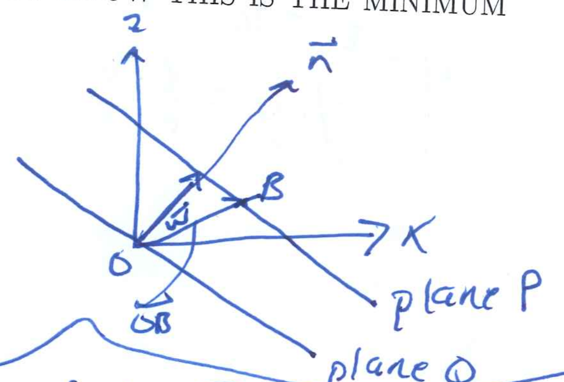
a. (*1 point*) Consider the plane $\mathcal{Q}$ given by the equation $x+y+z=0$ and the plane $\mathcal{P}$ given by the equation $x+y+z=1$. What can you say about the points of intersection of planes $\mathcal{P}$ and $\mathcal{Q}$? EXPLAIN YOUR ANSWER.

Planes P and Q must be parallel since they have the same normal vector, $\vec{n}=(1,1,1)$

Thus they have NO points of intersection

c. (*3 points*) What is the minimum distance between the planes $\mathcal{P}$ and $\mathcal{Q}$? SHOW ALL YOUR WORK AND GIVE AN EXPLANATION FOR HOW YOU KNOW THIS IS THE MINIMUM DISTANCE BETWEEN THE PLANES.

$\vec{x}=\begin{pmatrix}1\\1\\1\end{pmatrix}t$ is the equation of the line in the direction of the normal to the plane



when this line intersects plane Q we know $x+y+z=1$

so $t+t+t=1$

$3t=1$

$t=1/3$

$(1/3, 1/3, 1/3)$ is the point on plane P

Pick point on P and project onto normal and take magnitude

$\vec{w}=\text{proj}_{\vec{n}}(\vec{OB})=\dfrac{\begin{pmatrix}1\\0\\0\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}}{\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}}\begin{pmatrix}1\\1\\1\end{pmatrix}=\dfrac{1}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}$

$\|\vec{w}\|=\dfrac{1}{\sqrt{3}}=\dfrac{\sqrt{3}}{3}$

Distance between planes is $\|(1/3, 1/3, 1/3)-(0,0,0)\|$

$=\|(1/3,1/3,1/3)\|$

$=\sqrt{\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2}=\sqrt{\frac{1}{3}}=\frac{\sqrt{3}}{3}$

since the planes are parallel, their distance is CONSTANT.

JS