Exam 1 (Re-Do) : Multivariable Calculus

Math 212 Spring 2015
Prof. Ron Buckmire

ASSIGNED: Wednesday March 4
DUE: Wednesday March 18 9:35am

Name:

Directions:

Read *all* problems first before answering any of them. This tests consists of three (3) problems (and a BONUS problem) on six (6) pages.

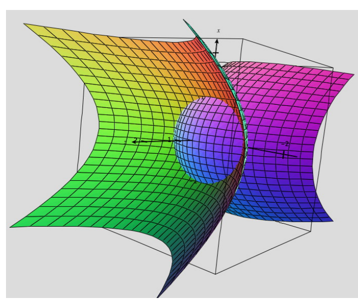
The topic of the problem is **in bold**, the number of points each problem is worth is in *italics* and the kind of skills required to solve each problem are in ALL CAPS.

This is a *voluntary reprise* of Midterm Exam 1*. This is a take-home version of that exam where you can take as long as you want until the exam is due. Points earned on this Exam will be combined with your points on the first Exam for your ultimate recorded grade on Exam 1.

You must show all relevant work to support your answers. Use complete English sentences to explain every answer and CLEARLY indicate your final answers to be graded from your "scratch work."

***You may consult a one-sided 8.5" by 11" "cheat sheet." Your original exam should be stapled to the front of this exam when you hand it in.**

Questions Policy: You may only speak about this take-home exam with me (Prof. Buckmire).



No.	Score	Maximum
1		30
2		40
3		30
BONUS		5
Total		**100**

Academic Ethics: I, ______________________________, attest that I have followed all the rules displayed above to the letter and in spirit and this exam reflects my own work conducted according to Occidental's Spirit of Honor.

1. (*30 points.*) ANALYTIC, COMPUTATIONAL, VERBAL. **Equation of Lines, Vector Operations, Parametric Equations.**

Lines in $\mathbb{R}^3$ which are neither parallel nor have any points of intersection are called **skew lines**. Consider the equations of two skew lines $\mathcal{L}_1$ and $\mathcal{L}_2$ in $\mathbb{R}^3$, given in parametric and vector form, respectively:

$$\mathcal{L}_1 : x = 1 + t, \quad y = 1 + t, \quad z = t \qquad \text{and} \qquad \mathcal{L}_2 : \vec{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$$

1(a) (*10 points.*) Show that the given lines are not **parallel** to each other. EXPLAIN YOUR ANSWER. (**HINT: vectors that are orthogonal to each other can not be parallel.**)

1(b) (*10 points.*) Show that the given lines **do not intersect**, i.e. they have no points in common. SHOW ALL YOUR WORK.

1(c) (*10 points.*) Write down equations representing three lines in $\mathbb{R}^3$ which are "skewed," i.e. they have no points in common with each other and are not parallel to each other. (**HINT: you may use $\mathcal{L}_1$ and $\mathcal{L}_2$ as two of your three skew lines if you wish but you do not have to.**) EXPLAIN HOW YOU KNOW YOUR THREE LINES ARE SKEWED.

2. (*40 points.*) ANALYTIC, COMPUTATIONAL, VISUAL. **Multivariable Functions, Cross-Sections, Level Sets, Tangent Planes, Gradient Vector.**
Consider the surface $z = f(x, y) = x - y^2$ for all part of this question.
2(a) (*8 points.*) Sketch graphs of the x-cross-section and y-cross-sections of this surface in the space below. LABEL YOUR AXES AND INDICATE WHICH CURVE REPRESENTS WHICH SLICE.

$z = f(x, y)$ when $x = c$ for $c = -1$, 0 or 1	$z = f(x, y)$ when $y = d$ for $d = -1$, 0 or 1

2(b) (*8 points.*) Compute $\vec{\nabla} f(3, -1)$.

2(c) (*8 points.*) Find the equation to the tangent plane to the surface $z = f(x, y) = x - y^2$ at the point $(3, -1)$. Make sure your equation is in the form $ax + by + cz + d = 0$. **HINT: What is normal vector to the surface at the point** $(3, -1)$**?**

2(d) (*8 points.*) Use your answer to **2(b)** to find the equation of the tangent line to the level set $f(x, y) = 2$ at the point $(3, -1)$. Make sure your tangent line equation is in the form $ax+by+c = 0$.
HINT: What is the slope of the tangent line to the level set $f(x, y) = 2$ at the point $(3, -1)$?

2(e) (*8 points.*) Recall $z = f(x, y) = x - y^2$. Sketch a graph of the level set $f(x, y) = 2$, the tangent line you found in part **2(d)** and the gradient vector you found in **2(b)** in the space below. LABEL YOUR FIGURE CAREFULLY!

Figure contains level set $f(x, y) = 2$, tangent line to this level set at $(3, -1)$ and $\vec{\nabla} f(3, -1)$

3. (*30 points.*) ANALYTIC, VISUAL, VERBAL. **Partial Derivatives, Multivariable Functions, Contour Diagrams, Directional Derivative.** Consider the following figure which depicts the contour diagram of an unknown function $f(x, y)$. The goal of this problem is for you to demonstrate your understanding of the relative rate of change of an unknown function $f(x, y)$ in specific directions using only information from its contour diagram.

3(a) (*9 points.*) Indicate the relative sizes of the partial derivatives of $f(x, y)$ at the points A, B and C in the contour diagram by writing the letters in the boxes below.

3(b) $\dfrac{\partial f}{\partial x}$

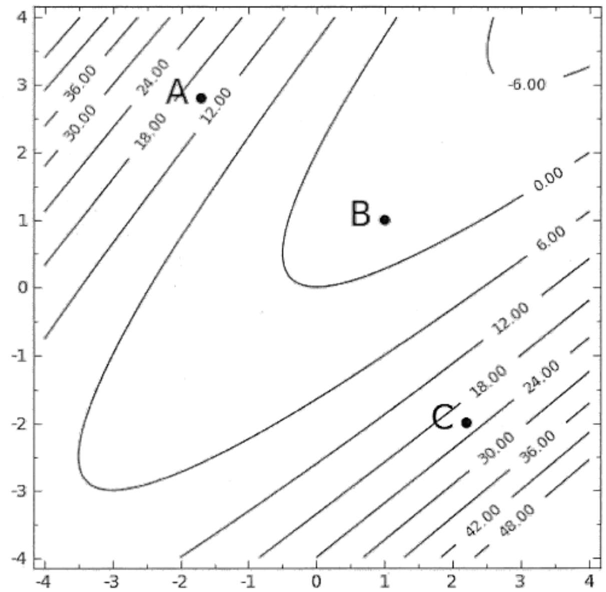
NEGATIVE [ ] < [ ] < [ ] **POSITIVE**

3(c) $\dfrac{\partial f}{\partial y}$

NEGATIVE [ ] < [ ] < [ ] **POSITIVE**

3(d) $f_{\vec{u}}$ where $\vec{u} = \hat{\imath} - \hat{\jmath}$

NEGATIVE [ ] < [ ] < [ ] **POSITIVE**



3(b) (*7 points.*) Discuss the reasoning for your ordering you gave above in **3(a)** of the relative sizes of the partial derivative of f with respect to x at the points A, B and C, i.e your placement of $f_x(A)$, $f_x(B)$ and $f_x(C)$ in increasing order in the boxes.

3(c) (*7 points.*) Discuss the reasoning for your ordering you gave above in **3(a)** of the relative sizes of the partial derivative of f with respect to y at the points A, B and C, i.e your placement of $f_y(A)$, $f_y(B)$ and $f_y(C)$ in increasing order in the boxes.

3(d) (*7 points.*) Discuss the reasoning for the ordering you gave above in **3(a)** of the relative sizes of the directional derivative of f in the direction $\vec{u} = \hat{\imath} - \hat{\jmath}$ at the points A, B and C, i.e your placement of $f_{\vec{u}}(A)$, $f_{\vec{u}}(B)$ and $f_{\vec{u}}(C)$ in increasing order in the boxes.

BONUS QUESTION. Vector Operations and Projection (*5 points*.)
Consider two parallel lines $\vec{x} = \vec{a} + \vec{d}t$ and $\vec{x} = \vec{b} + \vec{d}t$ in $\mathbb{R}^3$ where $\vec{a} \neq \vec{b}$. Obtain an expression for the minimum distance between these parallel lines. You may assume that $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$. Explain in words how you would determine the distance between the two lines and then write down a mathematical expression involving the given parameters that would allow this distance to be calculated for a specific example. Assume that none of the given vectors $\vec{a}$, $\vec{b}$ or $\vec{d}$ are the zero vector.
EXPLAIN YOUR ANSWER THOROUGHLY AND SHOW ALL YOUR WORK, EXTRA CREDIT POINTS ARE HARD TO EARN!