MATH 212 FALL 2014
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Study Guide for Exam II

(Mostly grabbed from Exam 2 Study Guide from Spring 2005 by Don Lawrence)

Exam 2 will cover material from Worksheets 11 through 21. I try to make study guides complete, but there is no guarantee – I might think of something later that I forgot to include. Also look at homeworks, quizzes, handouts, and class notes.

Topics

- Optimization
    - Locate and classify critical points in a contour diagram
    - Find critical points given a formula for f
    - Second derivative test
    - Applied max-min (you aren't required to classify your critical point in these)
    - Parametrize a constraint boundary
    - Lagrange multipliers
        - Find Lagrange points on a constraint
        - Understand basically why setting $\nabla f = \lambda \nabla g$ finds candidates
    - Extreme value theorem, including understanding what "closed" and "bounded" mean
    - Use limits to discuss the existence of global maxes/mins
- Integration
    - Predict the sign of a multiple integral
    - Compute a multiple integral
    - Sketch the region of integration
    - Choose or change the order of integration
    - Cylindrical coordinates and triple integrals
    - Spherical coordinates and triple integrals
- Alternative coordinates for integrals
    - Change integral between polar and Cartesian coordinates
    - Change integrals between Cartesian and spherical or cylindrical coordinates
- Chain rule
    - Chain rule diagrams
- Clairault's Theorem (Mixed partials of continuous functions are equal, or $f_{xy} = f_{yx}$)
- Graphical and algebraic computation of repeated partial derivatives like f_{xy}, f_{xx}, f_{yy}

Problems (Of course there are many other relevant problems in your text.) (Problems 1-4 were deleted)

(5) Find the critical points of $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$, and use the second derivative test to classify them as max, min, saddle, or can't tell. Then argue that f can't have a global max or min.

(6) $\int_0^1 \int_{2x}^2 \cos(1+y^2)\,dy\,dx =$

(7) *Set up* a multiple integral for finding the volume bounded by the plane $z = 6 - 2x - 3y$ and the coordinate planes. Do not *compute* the integral. [Hint: start by finding the line where $z = 6 - 2x - 3y$ intersects the xy-plane.]

(8) Find the global maximum and minimum of $f(x,y) = 3x - 2y$ on the ellipse $x^2 + 2y^2 = 44$. Name any methods and/or theorems you use.

(9) $\int_0^1 \int_{-2}^0 \int_{-\sqrt{4-y^2}}^0 \cos(x^2 + y^2)\,dx\,dy\,dz =$

(10) [10 points] Use limits to discuss the existence of global maxima and minima of $f(x,y) = x^2 + xy^4$.

(11) From the 1998 senior comps exam: Suppose $f(x,y,z) = x^2 + y^3 + 5yz$, and suppose $g(t) = (a(t), b(t), c(t))$ is a "vector-valued function". Suppose $a(0) = 4$, $b(0) = 5$, $c(0) = 6$, $a'(0) = 3$, $b'(0) = 2$, and $c'(0) = 1$. Let $h(t) = f(g(t))$. Find $h'(0)$.

(12) Draw the following contour diagram in the first quadrant: five parallel lines with negative slope, getting farther apart as they move away from the origin, labeled 5, 4, 3, 2, and 1 as you move away from the origin. Find the signs of the first and second partial derivatives.

(13) Suppose that $f(x,y)$ has continuous partials of all orders, and suppose that $f_{xxy} = x^2 y$, $f_{xyy} = xy^2$, and $f_{yyy} = y^3$.

(a) Find f_{yxxx}.

(b) Argue that there can't actually be such a function f.

(14) Write down the iterated integrals with correct limits which would be necessary to compute the volume of each wedge of cheese which has a perfectly horizontal base and height but that exactly six wedges will make up a completely circular cheese slab of radius *a* and height *h*.

(a) **Now change the order of integration and show that the volume is the same.**

(b) **Come up with *another* order of integration which represents the same volume.**

(c) **How many times can you do this?**

Math 212 Fall 2014 Blue Notes