Exam 1 (Re-Do): Multivariable Calculus

Math 212 Fall 2014 ASSIGNED: 11:45am Monday October 5
Prof. Ron Buckmire DUE: 11:45am Friday October 10

Name:

Directions:

Read *all* problems first before answering any of them. This tests consists of three (3) problems (and a BONUS problem) on six (6) pages.

The topic of the problem is **in bold**, the number of points each problem is worth is in *italics* and the kind of skills required to solve each problem are in ALL CAPS.

This is an **optional** retake of Exam 1 as a 4-day, open notes, open book, test. **No calculators or electronic devices may be used.**

You must show all relevant work to support your answers. Use complete English sentences as much as possible and CLEARLY indicate your final answers to be graded from your "scratch work."

Grading Policy: Your score on this Take-Home version of Exam 1 will be averaged with your in-class Exam 1 score to produce your official score on Exam 1.

Rules of Engagement: You may not discuss this take-home exam with any other students. You may ask questions of any professor. Your work on this exam must be solely your own.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

No.	Score	Maximum
1		30
2		30
3		40
BONUS		10
Total		**100**

1. (*30 points.*) ANALYTIC, COMPUTATIONAL, VISUAL. **Equation of Planes, Vector Operations.**

1(a) (*15 points*.) Find the general equation of the plane $\mathcal{P}$ in $\mathbb{R}^3$ that contains all three points $A(1,0,0)$, $B(1,2,-2)$, and $C(0,-3,4)$. ALSO EXPLAIN IN WORDS YOUR TECHNIQUE FOR DETERMINING THE EQUATION OF THE PLANE $\mathcal{P}$.

1(b) (*15 points*.) What is the minimum distance between the plane $\mathcal{P}$ found in part **1(a)** and the plane $\mathcal{Q}$ given by the equation $x+y+z=0$? ALSO EXPLAIN IN WORDS YOUR TECHNIQUE FOR DETERMINING THE MINIMUM DISTANCE BETWEEN PLANES $\mathcal{P}$ AND $\mathcal{Q}$.

2. (*30 points*.) VERBAL, ANALYTIC, COMPUTATIONAL. **Gradient, Directional Derivative, Tangent Planes**. Consider the following information about a mystery function $M(x, y, z)$:

(1) $M(x, y, z)$ increases most rapidly in the direction $\hat{v} = \frac{1}{7}(3\hat{i}+6\hat{j}+2\hat{k})$ at the point P$(-4, 2, 4)$.

(2) At the point P$(-4, 2, 4)$ the maximum rate of change of $M(x, y, z)$ is exactly 5.

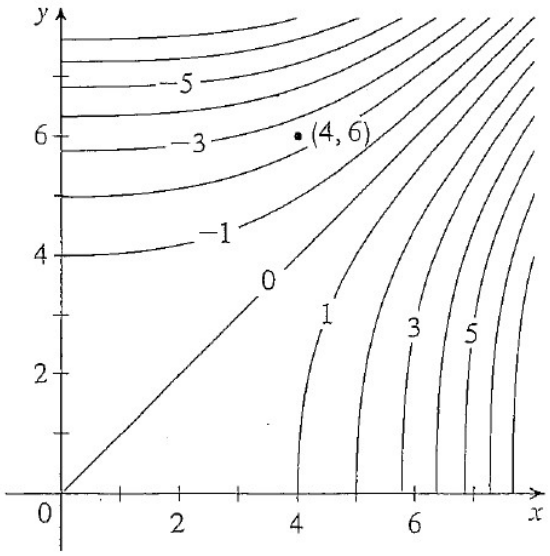
For each of the following questions, try to do the requested calculation or explain what missing information is necessary in order to do the calculation and how you would do the requested calculation if you had that information.

2(a) (*10 points*.) What is $M_{\vec{v}}(-4, 2, 4)$, the rate of change of $M(x, y, z)$ at the point P$(-4, 2, 4)$ in the direction $\vec{v} = 4\hat{i} - 3\hat{k}$?

2(b) (*10 points*.) In what direction $\vec{u}$ does the mystery function $M(x, y, z)$ change the least in magnitude at the point $(-4, 2, 4)$, i.e. in what direction parallel to $\vec{u}$ is $M_{\vec{u}}(-4, 2, 4)$ exactly zero? IF THERE IS MORE THAN ONE SUCH DIRECTION, EXPLAIN WHY AND STATE HOW MANY DIRECTIONS $M_{\vec{u}}(-4, 2, 4) = 0$.

2(c) (*10 points*.) What is the general equation of the tangent plane to the mystery function $M(x, y, z)$ at the point P$(-4, 2, 4)$?

3. (*40 points*.) ANALYTIC, VISUAL, VERBAL. **Partial Derivatives, Multivariable Functions, Cross-sections, Level Sets.** Consider the following figure which depicts the contour diagram of an unknown function $f(x, y)$. **NOTE:** $f(x, y) = 0$ **when** $y = x$**.**



3(a) (*10 points*.) What are the signs of $f_x(4, 6)$ and $f_y(4, 6)$? EXPLAIN YOUR ANSWER.

3(b) (*10 points*.) Indicate on the figure above a location (a, b) in the xy-plane where $f_x(a, b)$ **and** $f_y(a, b)$ will both be greater than they are at $(4, 6)$. Give the approximate coordinates of this point (if it exists). EXPLAIN YOUR ANSWER.

3(c) (*20 points.*) Again consider the contour diagram of the unknown function $f(x, y)$ given on the previous page.

In the space below give an accurate sketch of the graphs of the cross-sections of the unknown function $f(x, y)$. Sketch the cross-section of $f(x, y)$ when $x = 4$ from $0 \leq y \leq 7$. Then on a different set of labeled axes, give the cross-section $f(x, y)$ when $y = 6$ from $0 \leq x \leq 7$.

In the rest of the space, give written explanations for important features of your graphs (i.e. any changes in sign, slope or concavity, location of any zeroes, maximum and minimum values).

BONUS QUESTION. ANALYTIC, VERBAL. **Limits, Continuity.** (*10 points.*)
Consider $g : \mathbb{R}^2 \to \mathbb{R}$ where

$$g(x,y) = \begin{cases} \dfrac{x^2y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ A, & (x,y) = (0,0) \end{cases}$$

If it is possible, choose a value for the unknown constant A which will make the function $g(x,y)$ continuous for every $(x,y) \in \mathbb{R}^2$.

EXPLAIN YOUR ANSWER THOROUGHLY AND SHOW ALL YOUR WORK, EXTRA CREDIT POINTS ARE HARD TO EARN!