Multivariable Calculus

Class 30: **Wednesday April 26**

SUMMARY Exam 3 Review
CURRENT READING Williamson & Trotter, §7.1, §7.2, §7.3,§7.4, §8.1, §8.4, §9.1

Polar, Cylindrical and Spherical Coordinates

Iterated Integration

Fubini's Theorem

Multiple Integration: Double Integrals and Triple Integrals

Jacobi's Theorem (Change of Variables Theorem)

Path Integrals, Fundamental Theorem of Line Integration

Divergence, Curl, Gradient

Green's Theorem

(YOURS:) ____________________________

9. *[20 points total.]* **Green's Theorem.**

By evaluating the line integral $\frac{1}{2}\oint_\Gamma x\ dy - y\ dx$ and applying Green's Theorem, we want to show that the area of an ellipse is πab where $2a$ and $2b$ are the lengths of the minor and major axes of the ellipse and Γ is the closed path in $\mathbb{R}^2$ traced out by the ellipse, which is centered about the origin.

(a) (*6 points.*) Write down a multiple integral which represents the area of the ellipse. (Draw a picture!) **DO NOT EVALUATE THE INTEGRAL!**

(b) (*10 points.*) Evaluate the line integral $\frac{1}{2}\oint_\Gamma x\ dy - y\ dx$ where Γ is the closed path traced out by the ellipse in the counter clockwise direction.

(c) (*4 points.*) Explain how your above work allows you to find the area of the ellipse to be πab.

10. *[20 points total.]* **Gradient Fields, Div, Grad and Curl.**

Consider the vector field $\vec{F}(\vec{x}) = F_1(x,y,z)\hat{\imath} + F_2(x,y,z)\hat{\jmath} + F_3(x,y,z)\hat{k}$ where $\vec{x}$ is in $\mathbb{R}^3$. Recall, all gradient fields have zero curl. Recall also, a symmetric matrix is one in which $A_{ij} = A_{ji}$, in other words the transpose of matrix A equals matrix A. For example, $\begin{bmatrix} 1 & 0 & -7 \\ 0 & 2 & 4 \\ -7 & 4 & 3 \end{bmatrix}$ is symmetric.

(a) (*10 points.*) Show that if $\vec{F}$ is a **gradient field**, then the Jacobian of $\vec{F}$ is a **symmetric matrix**.

(b) (*10 points.*) Show that if the Jacobian of $\vec{F}$ is a **symmetric matrix**, then $\vec{\nabla} \times \vec{F} = \vec{0}$.