Multivariable Calculus

Math 212 Spring 2006 — Fowler 112 MWF 8:30pm - 9:25am

http://faculty.oxy.edu/ron/math/212/06/

Class 16: **Wednesday March 1**

SUMMARY Multivariable Newton's Method
CURRENT READING Williamson & Trotter, Section (Section 5.5)
HOMEWORK Williamson & Trotter, page 250: **1,4**;

Many engineering problems can be represented mathematically as either $A\vec{x} = \vec{b}$ or $\vec{f}(\vec{x} = \vec{0}$. There are even simple applications which end up involving the solution of $f(x) = 0$.

Newton's Method

Recall that Newton's Method is an algorithm for producing a sequence of approximations whose limit is the root of a function $f(x)$.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad x_0 \text{ given}$$

Newton's Method is derived from re-arranging the equation of the tangent line to $f(x)$ at the point x_n.

EXAMPLE

Show that the Babylonian Algorithm $x_{n+1} = \frac{1}{2}(x_n + \frac{A}{x_n})$ results from applying Newton's Method to find the root of $f(x) = x^2 - A$. Set $A = 5$ and $x_0 = 1$. Produce the sequence of approximations to $\sqrt{5}$.

The derivation of the **Multivariable Newton's Method** is very similar to the scalar version. The equation of the tangent approximation to the vector function of a vector variable $\vec{f}(\vec{x})$ is

$$\vec{T}(\vec{x}) = \vec{f}(\vec{x}_0) + J(\vec{x}_0)(\vec{x} - \vec{x}_0)$$

Let $\vec{x}_1$ have the property that $\vec{T}(\vec{x}_1) = \vec{0}$ and then solve for $\vec{x}_1$ to produce the result:

Multivariable Newton's Method

$$\vec{x}_{n+1} = \vec{x}_n - [J(\vec{x}_n)]^{-1}\vec{f}(\vec{x}_n), \qquad \vec{x}_0 \text{ given}$$

EXERCISE

Williamson & Trotter, page 250, #6. Let $\vec{g}(u,v) = \begin{bmatrix} u^2 + uv^2 \\ u + v^3 \end{bmatrix}$. Note that $\vec{g}(1,1) = (2,2)$. Use Newton's Method to approximate a solution to $\vec{g}(u,v) = (1.9, 2.1)$

Very often the Jacobian is only computed once (since it's a computationally expensive operation) and then NOT UPDATED for subsequent iterations. This kind of method is called a **quasi-Newton Method**.