# Multivariable Calculus

Math 212 Spring 2006 — Fowler 112 MWF 8:30pm - 9:25am

 — http://faculty.oxy.edu/ron/math/212/06/

*Class 10*: **Monday February 13**

**SUMMARY** Partial Derivatives
**CURRENT READING** Williamson & Trotter, Section 4.3
**HOMEWORK** Williamson & Trotter, page 203: 3, **9, 12**, 22, 25, 31, **34**, 37;
**Extra Credit page 204: # 38, 39**

DEFINITION

The **partial derivative** of a scalar function of a vector variable $f(\vec{x})$ where $f : \mathbb{R}^n \to \mathbb{R}$ can be defined as

$$\frac{\partial f}{\partial x_i} = \lim_{h\to 0} \frac{f(x_1, x_2, x_i + h, \ldots, x_n) - f(x_1, x_2, \ldots, x_n)}{h}$$

In practice, this basically means that if you have a function of many variables when you take a partial derivative with respect to a particular variable you treat all the other variables in the function as CONSTANTS.

**NOTE:** Sometimes $\dfrac{\partial f}{\partial x}$ will be denoted simply $f_x$.

EXAMPLE 1

Consider $f(x, y, z) = xyz + \sin(x + y) + z^2y + e^{-x}$ and calculate $f_x$, $f_y$ and $f_z$

**Exercise 1**

Find $\dfrac{\partial^3 f}{\partial x^2 \partial y}$ (also known as $f_{xxy}$) if $f(x, y) = \ln(2x + 3y)$

**Equation of a Tangent Plane to a Surface**
The equation of a tangent plane to a surface $z = f(x, y)$ at the point $(a, b, f(a, b))$ is given by the equation

$$z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

**Exercise 2**

**Williamson & Trotter, page 204, page 10**. Find the equation of the tangent plane to $f(x, y) = x(y^2 + 1)$ at $(a, b) = (0, 2)$.

**Graphical Interpretation of Tangent Plane**
The notion of the existence of a tangent plane to a surface is the 3-dimension equivalent to the existence of a tangent line or **local linearity** of a curve in 2-dimensions. The existence of these objects relate to the **differentiability** of the scalar function $f(\vec{x})$, and **differentiability of a function at a point implies continuity at that point**, but continuity DOES NOT imply differentiability.

**Connection to Taylor's Theorem and Miscroscope Approximation**
We can approximate a function $f(x, y)$ near the point $(x_0, y_0)$ with its tangent plane:
$f(x, y) \approx f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

EXAMPLE 2

Approximate the value of $f(0.97, 2.01)$ where $f(x, y) = \sqrt{x^2 + y^3}$.

**Clairault's Theorem**

If $f : \mathbb{R}^2 \to \mathbb{R}$ is continuous and $f_x, f_y, f_{xy}$ and $f_{yx}$ are also continuous on the same domain as $f$, then $f_{xy} = f_{yx}$. **NOTE**: $f_{xy} = (f_x)_y$.

**Exercise 3**

Show that Clairault's Theorem applies to the function $f(x, y) = x^y$ by proving $f_{xy} = f_{yx} = (1 + y \ln x)x^{y-1}$.