Multivariable Calculus

Class 5: **Wednesday February 1**

SUMMARY Review of Linear Systems, Part 1: An Overview
CURRENT READING Williamson & Trotter, Section 2.1
HOMEWORK Williamson & Trotter, page 44-45: 1,14,17,19,20,24,31,32,33,34,35;

System of Linear Equations

Linear systems of equations appear in a wide variety of applications, from fluid dynamics to probability theory and bioinformatics. Specifically in multivariable calculus one often has to solve linear system in order to find the point of intersection of a number of objects.

Recall that a system of linear equations

$$\begin{aligned} a_{11}x + a_{12}y + a_{13}z &= b_1 \\ a_{21}x + a_{22}y + a_{23}z &= b_2 \\ a_{31}x + a_{32}y + a_{33}z &= b_3 \end{aligned}$$

can also be written as matrix system $A\vec{x} = \vec{b}$ where A is called the **coefficient matrix** and $\vec{b}$ is the right-hand side and $\vec{x}$ is the solution vector.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \text{ and } \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

This is known as the **matrix view**. Notice that the rows of the matrix correspond to the coefficients in the original linear equations. This is known as the **row view**. You can also think of the linear system as a question of whether a **linear combination** of vectors $\vec{a}_1$, $\vec{a}_2$ and $\vec{a}_3$ will equal the given vector $\vec{b}$

$$\begin{aligned} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix} x + \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} y + \begin{bmatrix} a_{31} \\ a_{32} \\ a_{33} \end{bmatrix} z &= \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \\ \vec{a}_1 x + \vec{a}_2 y + \vec{a}_3 z &= \vec{b} \end{aligned}$$

Notice that the columns of the coefficient matrix correspond to the vectors $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$. This is called the **column view**.

EXERCISE 1

Consider $x + y = 1$ and $x - y = 2$. Find the point(s) of intersection in $\mathbb{R}^2$ (if they exist).

EXERCISE 2

Now consider $x+y+z=0$, $x-y=0$ along with $y+z=0$. Find the point(s) of intersection in $\mathbb{R}^3$ (if they exist).

What are the geometrical interpretations of your answers in Exercise 1 and Exercise 2?

EXERCISE 3

Is $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$?

EXERCISE 4

What is the solution of $\begin{bmatrix} 2 & -2 & 2 \\ -1 & 8 & 1 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -1 \end{bmatrix}$?

Classification of Linear Systems

A linear system $A\vec{x} = \vec{b}$ can either be **singular** (i.e. not have a unique solution or have no solution at all) or **non-singular** (have a unique solution).

GROUPWORK

Write down examples of linear systems in $\mathbb{R}^2$ which represent the three different kinds of possible result: i.e. **0, 1 or ∞ solution(s).**