Math 212 Spring 2006

BONUS Quiz **3** — **Multivariable Calculus**

Name: ______________________________

Date: ____________________ **Friday February 10**
Time Begun: ____________________ Ron Buckmire
Time Ended: ____________________

Topic : Determinants

To see if you can synthesize your understanding of determinants to obtain an interesting result.

Reality Check:

EXPECTED SCORE : __________**/10** ACTUAL SCORE : __________**/10**

Instructions:

0. Please look for a hint on this quiz posted to `faculty.oxy.edu/ron/math/212/06/`.
1. Once you open the quiz, you have as much time as you like to complete it, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday February 13**, in class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Question #6, Math 214 Final Exam, Fall 2003. Suppose the n by n matrix A_n has 3's along its main diagonal and 2's along the diagonal below and in the $[1, n]$ position. There are zeroes everywhere else. The goal of this question is to obtain a formula for the determinant of A_n (obviously it will depend on n).

$$A_2 = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \; A_3 = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 3 & 0 \\ 0 & 2 & 3 \end{bmatrix} \; A_4 = \begin{bmatrix} 3 & 0 & 0 & 2 \\ 2 & 3 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & 2 & 3 \end{bmatrix} \; \ldots \quad A_n = \begin{bmatrix} 3 & 0 & \ldots & 0 & 2 \\ 2 & 3 & 0 & \ldots & 0 \\ 0 & 2 & 3 & \ddots & 0 \\ \vdots & 0 & \ddots & \ddots & 0 \\ 0 & 0 & \ldots & 2 & 3 \end{bmatrix}$$

a. (*4 points*). Find the determinant of A_2, A_3 and A_4. [**HINT: Use the first row of A_n!**]

b. (*4 points*). Using your answers in part **(a)** you should now be able to hypothesize a formula for $\det(A_n)$. Check that your formula for $\det(A_n)$ works for A_2, A_3, A_4 and predict the determinant of A_5

c. (*2 points*). Check that the actual determinant of A_5 equals that predicted by your formula.