(Make-Up) Test 1: Multivariable Calculus

Assigned: Fri Mar 6 | Due: Wed Mar 22

Ron Buckmire | Math 212 Spring 2006

Name: ____________________

Directions: Read *all* problems first before answering any of them. There are 6 pages in this test. This is a make-up version of Test 1. You may consult whatever notes and books you like. **No calculators.** You must show all relevant work to support your answers. Use complete English sentences and CLEARLY indicate your final answers to be graded from your "scratch work."

You may not talk to any other human being about the test except for Professor Ron Buckmire. In addition, you must have had a conference (meeting) with him in order to be eligible to take this make-up test.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

No.	Score	Maximum
1		30
2		40
3		30
BONUS		5
Total		**100**

1. Multivariable Functions, Differentiation, Point Sets. *30 points.*

TRUE or FALSE – put your answer in the box (1 point). To receive FULL credit, you must also give a brief, and correct, explanation in support of your answer! Remember if you think a statement is TRUE you must prove it is ALWAYS true. If you think a statement is FALSE then all you have to do is show there exists a counterexample which proves the statement is FALSE at least once. .

(a) *10 points.* **TRUE or FALSE?** "There does not exist a vector function of a scalar variable whose derivative equals itself."

(b) *10 points.* **TRUE or FALSE?** "There does not exist a scalar function of a vector variable whose gradient equals itself."

(c) *10 points.* **TRUE or FALSE?** "There does not exist a point set in $\mathbb{R}^n$ which is both open and closed."

2. Analytic Geometry in $\mathbb{R}^n$, Lines, Planes. Recall that a hyperplane in $\mathbb{R}^n$ is represented by an equation of the form $\sum_{k=1}^{n} c_k x_k = c_0$. This problem is about the geometric objects created by the intersection of two hyperplanes in $\mathbb{R}^n$.

(a) (*10 points.*) Consider the following two hyperplanes in $\mathbb{R}^2$:

$$\begin{aligned} x + y &= 1 \\ x - y &= 1 \end{aligned}$$

Do there exist points in $\mathbb{R}^2$ that lie on both of these objects simultaneously? If so, find all such points of intersection (i.e. a vector equation in $\mathbb{R}^2$ describing the object which is made up of these special points) and describe the object (i.e. name it) and give the dimension of the object (i.e. zero-dimensional, one-dimensional, two-dimensional, three-dimensional, et cetera).

(b) (*10 points.*) Consider the following two hyperplanes in $\mathbb{R}^3$:

$$\begin{aligned} x + y + z &= 1 \\ x + y - z &= 1 \end{aligned}$$

Do there exist points in $\mathbb{R}^3$ that lie on both of these objects simultaneously? If so, find all such points of intersection (i.e. a vector equation in $\mathbb{R}^3$ describing the object which is made up of these special points) and describe the object (i.e. name it) and give the dimension of the object (i.e. zero-dimensional, one-dimensional, two-dimensional, three-dimensional, et cetera).

(c) (*10 points.*) Consider the following two hyperplanes in $\mathbb{R}^4$:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 1 \\ x_1 + x_2 + x_3 - x_4 &= 1 \end{aligned}$$

Do there exist points in $\mathbb{R}^4$ that lie on both of these objects simultaneously? If so, find all such points of intersection (i.e. a vector equation in $\mathbb{R}^4$ describing the object which is made up of these special points) and describe the object (i.e. name it) and give the dimension of the object (i.e. zero-dimensional, one-dimensional, two-dimensional, three-dimensional, et cetera).

(d) (*10 points.*) (**The point!**) Consider the following two hyperplanes in $\mathbb{R}^n$:

$$\begin{aligned} x_1 + x_2 + x_3 + \ldots + x_{n-1} + x_n &= 1 \\ x_1 + x_2 + x_3 + \ldots + x_{n-1} - x_n &= 1 \end{aligned}$$

Do there exist points in $\mathbb{R}^n$ that lie on both of these objects simultaneously? If so, find all such points of intersection (i.e. a vector equation in $\mathbb{R}^n$ describing the object which is made up of these special points) and describe the object (i.e. name it) and give the dimension of the object (i.e. zero-dimensional, one-dimensional, two-dimensional, three-dimensional, et cetera).

3. Partial Derivatives. *30 points.*

The energy E, of a body of mass m moving with speed $v \geq 0$ is given by the formula

$$E = mc^2 \left(\frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} - 1 \right)$$

where c is the speed of light.

(a) (*15 points.*) Compute $\dfrac{\partial E}{\partial m}$. What do you expect the sign of $\dfrac{\partial E}{\partial m}$ to be? **EXPLAIN YOUR ANSWER.** (Write down a few sentences explaining the physical interpretation of your answer.)

(b) (*15 points.*) Compute $\dfrac{\partial E}{\partial v}$ What do you expect the sign of $\dfrac{\partial E}{\partial v}$ to be? **EXPLAIN YOUR ANSWER.** (Write down a few sentences explaining the physical interpretation of your answer.)

BONUS QUESTION. Vectors, Dot Product, Magnitude. (*5 points.*)
Consider the following statements.

Let $\vec{u}$ be a vector such that $|\vec{u}| = 1$. Choose a vector $\vec{v}$ such that $\vec{u} \cdot \vec{v} = 3$ and $|\vec{v}| = \sqrt{5}$.

$$\begin{aligned} |\vec{u} - \vec{v}|^2 &= (\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v}) \\ &= \vec{u} \cdot \vec{u} - 2\vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{v} \\ &= 1 - 2(3) + 5 \\ &= 0 \end{aligned}$$

This implies that $\vec{u} = \vec{v}$ since $\vec{u} - \vec{v} = \vec{0}$. But $\vec{u}$ and $\vec{v}$ have different lengths.

HOW CAN TWO VECTORS BE EQUAL YET HAVE DIFFERENT LENGTHS? FIND THE ERROR! EXPLAIN YOUR ANSWER THOROUGHLY, EXTRA CREDIT POINTS ARE HARD TO GET.