# Introduction to Grades Eight Through Twelve

The standards for grades eight through twelve are organized differently from those for kindergarten through grade seven. In this section strands are not used for organizational purposes as they are in the elementary grades because the mathematics studied in grades eight through twelve falls naturally under discipline headings: algebra, geometry, and so forth. Many schools teach this material in traditional courses; others teach it in an integrated fashion. To allow local educational agencies and teachers flexibility in teaching the material, the standards for grades eight through twelve do not mandate that a particular discipline be initiated and completed in a single grade. The core content of these subjects must be covered; students are expected to achieve the standards however these subjects are sequenced.

Standards are provided for Algebra I, geometry, Algebra II, trigonometry, mathematical analysis, linear algebra, probability and statistics, advanced placement probability and statistics, and calculus. Many of the more advanced subjects are not taught in every middle school or high school. Moreover, schools and districts have different ways of combining the subject matter in these various disciplines. For example, many schools combine some trigonometry, mathematical analysis, and linear algebra to form a precalculus course. Some districts prefer offering trigonometry content with Algebra II.

Table 1, "Mathematics Disciplines, by Grade Level," reflects typical grade-level groupings of these disciplines in both integrated and traditional curricula. The lightly shaded region reflects the minimum requirement for mastery by all students. The dark shaded region depicts content that is typically considered elective but that should also be mastered by students who complete the other disciplines in the lower grade levels and continue the study of mathematics.

Many other combinations of these advanced subjects into courses are possible. What is described in this section are standards for the academic content by discipline; this document does not endorse a particular choice of structure for courses or a particular method of teaching the mathematical content.

When students delve deeply into mathematics, they gain not only conceptual understanding of mathematical principles but also knowledge of, and experience with, pure reasoning. One of the most important goals of mathematics is to teach students logical reasoning. The logical reasoning inherent in the study of mathematics allows for applications to a broad range of situations in which answers to practical problems can be found with accuracy.

By grade eight, students' mathematical sensitivity should be sharpened. Students need to start perceiving logical subtleties and appreciate the need for sound mathematical arguments before making conclusions. Students who are not prepared for Algebra I by grade nine should instead receive specialized instructional materials that focus on the prerequisite standards described in Appendix E. An algebra readiness course will prepare students for success in algebra and subsequent advanced courses. As students progress in the study of mathematics, they learn to distinguish between inductive and deductive reasoning; understand

**Table 1. Mathematics Disciplines, by Grade Level**

| *Disciplines* | *Grades* | | | | |
|---|---|---|---|---|---|
| | *Eight* | *Nine* | *Ten* | *Eleven* | *Twelve* |
| Algebra I | | | | | |
| Geometry | | | | | |
| Algebra II | | | | | |
| Probability and Statistics | | | | | |
| Trigonometry | | | | | |
| Linear Algebra | | | | | |
| Mathematical Analysis | | | | | |
| Advanced Placement Probability and Statistics | | | | | |
| Calculus | | | | | |

the meaning of logical implication; test general assertions; realize that one counterexample is enough to show that a general assertion is false; understand conceptually that although a general assertion is true in a few cases, it may not be true in all cases; distinguish between something being proven and a mere plausibility argument; and identify logical errors in chains of reasoning.

Mathematical reasoning and conceptual understanding are not separate from content; they are intrinsic to the mathematical discipline students master at more advanced levels.

# Algebra I Mathematics Content Standards

Symbolic reasoning and calculations with symbols are central in algebra. Through the study of algebra, a student develops an understanding of the symbolic language of mathematics and the sciences. In addition, algebraic skills and concepts are developed and used in a wide variety of problem-solving situations.

**Note:** The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified before they are used with students.

**1.0** Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable:

1.1 Students use properties of numbers to demonstrate whether assertions are true or false.

**2.0** Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

Simplify $\left(x^3 y^{\frac{1}{2}}\right)^6 \sqrt{xy}$.

**3.0** Students solve equations and inequalities involving absolute values.

Solve for $x$: $3|x| + 5 > 7$.

For which values of $x$ is $|x+4| = |x| + 4$?

**4.0** Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x - 5) + 4(x - 2) = 12$.

For what values of x is the following inequality valid?

$5(x - 1) > 3x + 2$.

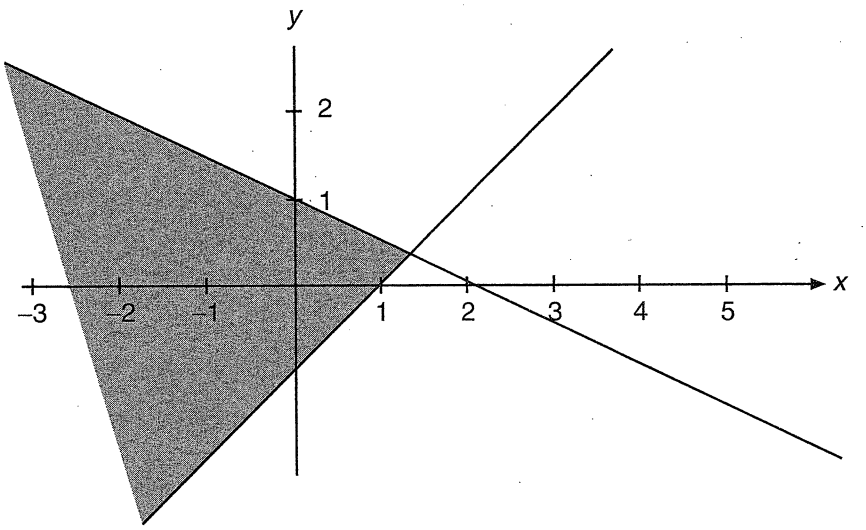
Expand and simplify $2(3x + 1) - 8x$.

**5.0** Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

A-1 Pager Company charges a $25 set-up fee plus a $6.50 monthly charge. Cheaper Beeper charges $8 per month with no set-up fee. Set up an inequality to determine how long one would need to have the pager until the A-1 Pager plan would be the less expensive one.

**6.0** Students graph a linear equation and compute the *x*- and *y*-intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequalities (e.g., they sketch the region defined by $2x + 6y < 4$).

Find inequalities whose simultaneous solution defines the region shown below:




**7.0** Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.

Does the point (1, 2) lie on, above, or below the graph of the line $3x - 5y + 8 = 0$? Explain how you can be sure of your answer.

Write the equation of the line having *x*-intercept $-2\frac{1}{3}$ and *y*-intercept 5.

**8.0** Students understand the concepts of parallel lines and perpendicular lines and how their slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

Find the equation of the line passing through $(-1, \frac{1}{3})$ and parallel to the line defined by $5x + 2y = 17$. Also find the equation of the line passing through the same point but perpendicular to the line $5x + 2y = 17$.

**9.0** Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

Solve and sketch the lines and the solution set:

$3x + y = -1$

$x - \frac{1}{2}y = \frac{4}{3}$

**Algebra I**

**10.0** Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.

**11.0** Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Factor $9x^3 + 6x^2 + x$.

**12.0** Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.

Simplify $\frac{x^2+2x+1}{x^2-1}$.

**13.0** Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.

Solve for $x$ and give a reason for each step: $\frac{2}{3x+1}+2=\frac{2}{3}$. (ICAS 1997, 6)

**14.0** Students solve a quadratic equation by factoring or completing the square.

**15.0** Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

The sum of the two digits of a number is 10. If 36 is added to it, the digits will be reversed. Find the number.

Two cars A and B move at constant velocity. Car A starts from $P$ to $Q$, 150 miles apart, at the same time that car B starts from $Q$ to $P$. They meet at the end of $1\frac{1}{2}$ hours. If car A moves 10 miles per hour faster than car B, what are their velocities?

**16.0** Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.

**17.0** Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

**18.0** Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

**19.0** Students know the quadratic formula and are familiar with its proof by completing the square.

Toni is solving this equation by completing the square.

$ax^2 + bx + c = 0$ (where $a \geq 0$)

Step 1. $ax^2 + bx = -c$

Step 2. $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Step 3. ?

Which response shown below should be step 3 in the solution?

1. $x^2 = -\frac{c}{b} - \frac{b}{a}x.$

2. $x + \frac{b}{a} = -\frac{c}{ax}.$

3. $x^2 + \frac{b}{a}x + \frac{b}{2a} = -\frac{c}{a} + \frac{b}{2a}.$

4. $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2.$

(CST released test question, 2004)



**20.0** Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.

Suppose the graph of $y = px^2 + 5x + 2$ intersects the $x$-axis at two distinct points, where $p$ is a constant. What are the possible values of $p$?

**21.0** Students graph quadratic functions and know that their roots are the $x$-intercepts.

The graph of $y = x^2 + bx - 1$ passes through $(-\frac{1}{3}, 0)$

What is $b$?

**22.0** Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the $x$-axis in zero, one, or two points.

**23.0** Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.

**Algebra I**

**24.0** Students use and know simple aspects of a logical argument:

24.1 Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.

24.2 Students identify the hypothesis and conclusion in logical deduction.

24.3 Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

**25.0** Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:

25.1 Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.

25.2 Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.

25.3 Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.