

Math 120 Spring 98

Quiz 7

Basic Calculus 2

Name: _____

Date: _____

Time Begun: _____

Time Ended: _____

Friday April 3, 1998

Ron Buckmire

Topic covered: Determining Convergence/Divergence of Improper Integrals by Comparison

The idea behind this quiz is to give you more practice determining whether a particular improper integral will converge or diverge without evaluating it, but by comparing it to an improper integral you already know something about.

Instructions:

1. Once you open the quiz, you have 90 minutes to complete it.
2. You may use the book or any of your class notes, and you may use a calculator. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Relax and enjoy..
6. **This quiz is due on Monday, April 6, in class. NO LATE QUIZZES WILL BE ACCEPTED.**

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Consider the function $f(z) = \frac{1}{\sqrt{z} + z} = \frac{1}{z^{1/2} + z}$. We want to try and determine whether

$I = \int_1^{\infty} \frac{1}{\sqrt{z} + z} dz$ and $J = \int_0^1 \frac{1}{\sqrt{z} + z} dz$ converge or diverge.

- (a) (4 points) Use the rules developed in class to say whether each of the following integrals converges or diverges:

$$\int_0^1 \frac{dz}{2\sqrt{z}}$$

$$\int_1^{\infty} \frac{dz}{2\sqrt{z}}$$

$$\int_0^1 \frac{dz}{2z}$$

$$\int_1^{\infty} \frac{dz}{2z}$$

- (b) (1 point) For $z > 1$ is $\frac{1}{\sqrt{z} + z} > \frac{1}{z + z}$ or $\frac{1}{\sqrt{z} + z} < \frac{1}{z + z}$?

For $z > 1$ is $\frac{1}{\sqrt{z} + z} > \frac{1}{\sqrt{z} + \sqrt{z}}$ or $\frac{1}{\sqrt{z} + z} < \frac{1}{\sqrt{z} + \sqrt{z}}$?

- (c) (2 points) Does $I = \int_1^{\infty} \frac{dz}{\sqrt{z} + z}$ converge or diverge? Support your answer by explaining how the inequalities in part (b) and your answers to part (a) allow you to determine whether I converges or diverges without actually evaluating it. If you like, you can use a graph to support your explanation.

- (d) (1 point) For $0 < z < 1$ is $\frac{1}{\sqrt{z} + z} > \frac{1}{z}$ or $\frac{1}{\sqrt{z} + z} < \frac{1}{z}$?

For $0 < z < 1$ is $\frac{1}{\sqrt{z} + z} > \frac{1}{\sqrt{z} + \sqrt{z}}$ or $\frac{1}{\sqrt{z} + z} < \frac{1}{\sqrt{z} + \sqrt{z}}$?

- (e) (2 points) Does $J = \int_0^1 \frac{dz}{\sqrt{z} + z}$ converge or diverge? Support your answer by explaining how the inequalities in part (d) and your answers to part (a) allow you to determine whether J converges or diverges without actually evaluating it. If you like, you can use a graph to support your explanation.