Calculus 2

Class 17: Friday March 7

Applications of Integration: Calculating Volumes Using Shells

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Warm-Up

(a) What is the surface area of an empty cylinder of radius r and height h (without lids)?

(b) What is the volume of an empty cylinder of radius r and height h which has a very small thickness Δr?

Calculating Volume Using the Shell Method

FORMULA: **Volume Using Shells**

The volume of a solid of revolution obtained by rotating the region under the curve$f(x)$ (i.e. bounded by $x = a$ and $x = b$ and the x-axis and $y = f(x)$) about the y-axis is given by

$$V = \int_a^b 2\pi\ x\ f(x)\ dx$$

Note: Instead of adding up circular slices (or disks) of width dx or dy with area $\pi \cdot \text{radius}^2$ which is what the washer or disk method does, the shell method is about adding up cylindrical shells with width dx and height $f(x)$ and circumference $2\pi x$ (when the volume is rotated about the y-axis). In other words,

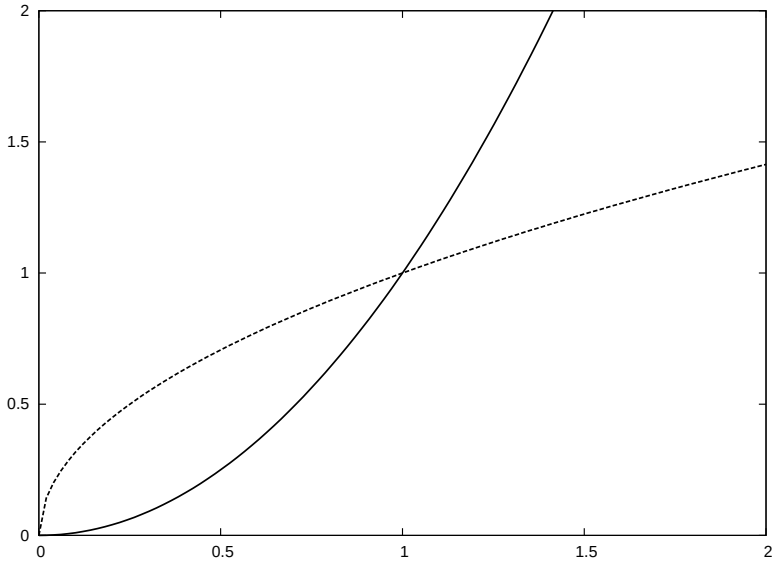
$$\text{Volume of Shell} = \text{Circumference } \cdot \text{Height} \cdot \text{Thickness}$$

Exercise

Let's show that we can find the volume of a solid cylinder of radius r and height h using the shell method by evaluating the integral $\displaystyle\int_0^r 2\pi r \cdot h \cdot dr$

EXAMPLE

Recall that the volume of the solid formed by rotating the region bounded by $y = x^3$, $y = 8$ and $x = 0$ about the y-axis is $\dfrac{96}{5}\pi$. Let's confirm that volume by using the method of shells.



Same Volume, Different Methods

Let's consider the volume of the solid of revolution formed by rotating the region bounded by $y = x^2$ and $y = \sqrt{x}$. We can compute this volume using shells or washers.

Web Resource

Paul Seeburger has a Java-based applet which allows one to visualize this volume using washers and shells. The URLs where this can be found are:

`http://web.monroecc.edu/manila/webfiles/calcNSF/JavaCode/other/myWasher1.htm`

and

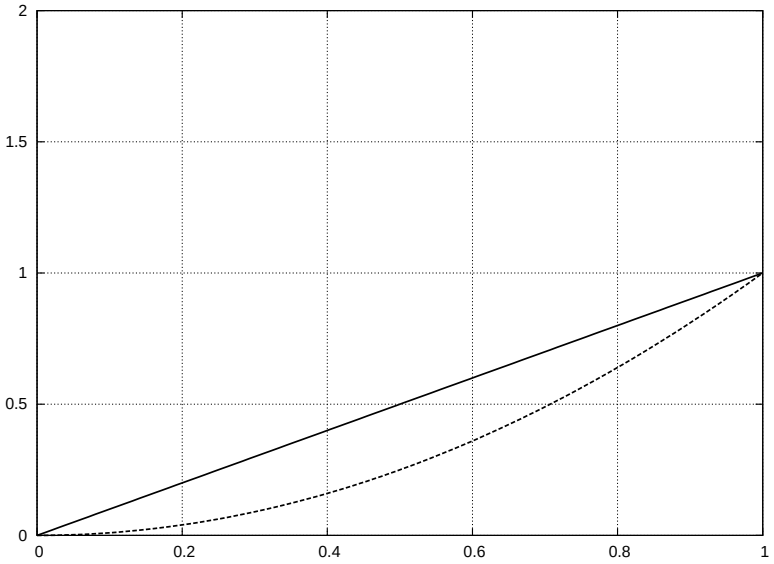
`http://web.monroecc.edu/manila/webfiles/calcNSF/JavaCode/other/myShell1.htm`

Check them out! (You may need to adjust your brower's Java settings for the animation to work.)

EXAMPLE

Let's find the volume of this solid formed by rotating the region bounded by $y = x^2$ and $y = \sqrt{x}$ about the y-axis using the method of shells.

Let's also find the volume of this solid formed by rotating the region bounded bounded by $y = x^2$ and $y = \sqrt{x}$ about the y-axis using the method of washers.



GroupWork

Recall in *Class 16* that we used the disk or washer method to find the volume of the solid formed by rotating the region between the curves $y = x$ and $y = x^2$ about the following lines. Repeat the calculation of these volumes using the method of shells.

(a) the x-axis; $V = \dfrac{2}{15}\pi$ **(b)** the y-axis; $V = \dfrac{\pi}{6}$ **(c)** the line y=2; $V = \dfrac{8}{15}\pi$