Class 14: Wednesday February 26

Applications of Integration: Area Between Curves

`Warm-Up`

What is the point of intersection of the curves $f(x) = \sqrt{2x}$ and $g(x) = 8x^3$?

Finding the Area between two curves

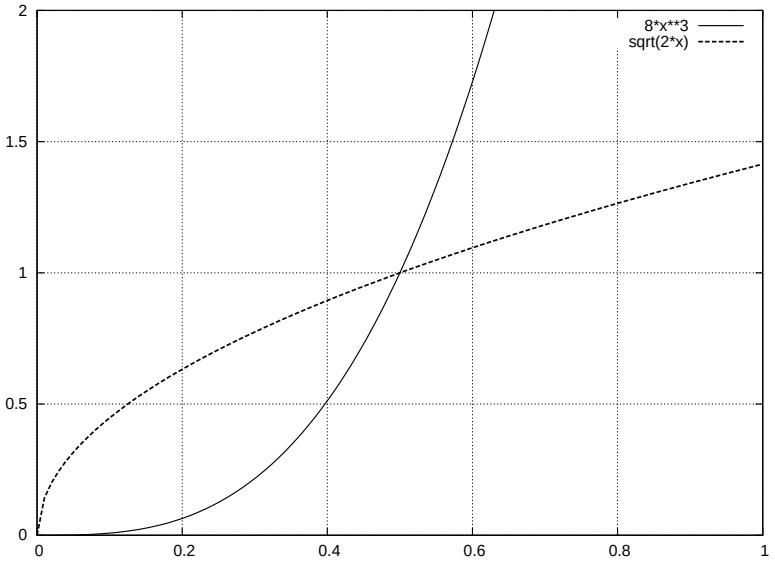
`TOP-BOTTOM FORMULA`

The area A bounded by two curves $y = f(x)$and $y = g(x)$ and two lines $x = a$ and $x = b$ where f and g are continuous and $f(x) \geq g(x)$ for all x on the interval $a \leq x \leq b$ is given by

$$A = \int_a^b [f(x) - g(x)]\, dx$$

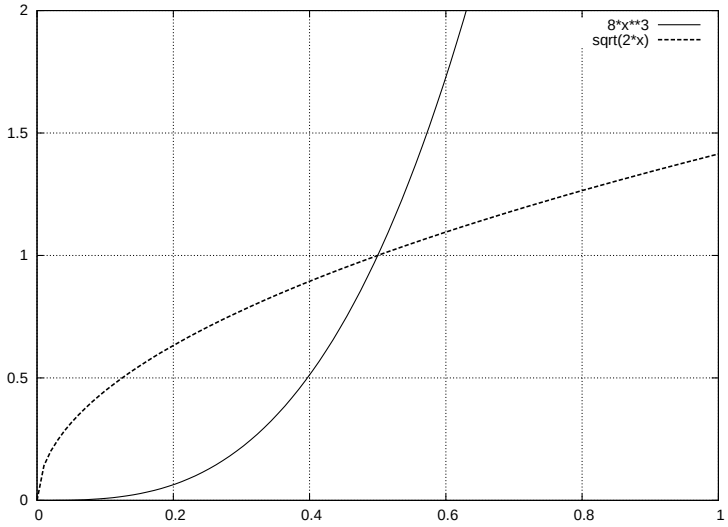
EXAMPLE

Look at the figure below and write down a definite integral which represents the value of the shaded area A. The area A represents the area between two curves, $f(x) = \sqrt{2x}$ and $g(x) = 8x^3$



Let's compute the value of A by using integration.

A Different Way Of Looking At The Same Shape



RIGHT-LEFT FORMULA

We can think of this shape as being bounded by two curves $x = L(y)$ and $x = R(y$ and the lines $y = c$ and $y = d$. In that case, the area A would be given by

$$A = \int_c^d [R(y) - L(y)]\, dy$$

Exercise

What are the functions $x = L(y)$, $x = R(y)$ and the lines $y = c$ and $y = d$ for the area above?

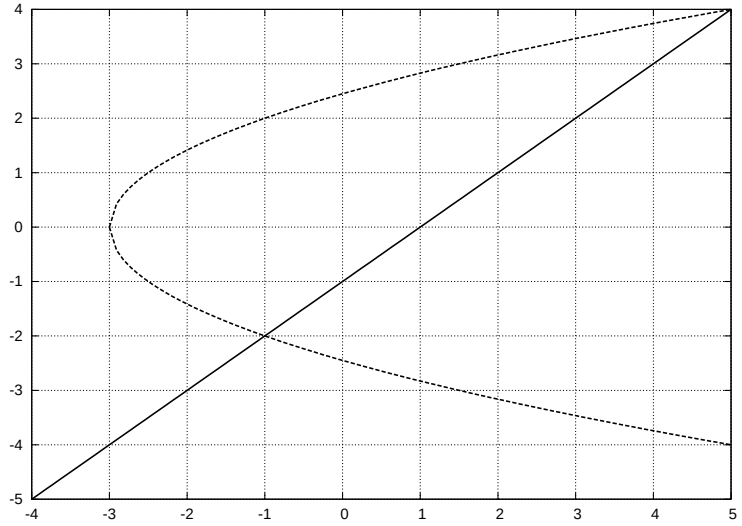
EXAMPLE

Compute the value of A again, this time using horizontal boxes.

Depending on the shape of particular area, you should choose horizontal boxes (i.e. a Right-Left dy integral) or vertical boxes (a Top-Bottom dx integral).

GROUPWORK

1. Find the area between the line $y = x - 1$ and the parabola $y^2 = 2x + 6$



2. **Stewart, page 369, #7.** Find the area between the curves $y = (x - 2)^2$ and $y = x$.