Class 9: Monday February 10

Partial Fractions

`Warm-Up`

(a) Find A and B so that $\dfrac{5}{12} = \dfrac{A}{4} + \dfrac{B}{3}$.

(b) What is the degree of the polynomial $A(x) = x^3 + x - 2$? If $A(1) = 0$ what does this tell you about $A(x)$?

RECALL

Fractions can show up with polynomial expressions as well. Suppose you have two polynomials $P(x)$ of degree p and a different polynomial $Q(x)$ of degree q then we call the expression $\dfrac{P(x)}{Q(x)}$ a rational expression. Today we're goingh to look at a techniques for how to find anti-derivatives of rational expressions.

Partial Fractions

Note, we are particularly interested in the cases where $q > p$. If we can factor the denominator polynomial $Q(x)$ as a product of linear terms so that $Q(x) = (a_1x + b_1)(a_2x + b_2)\dots(a_qx + b_q)$ then
Partial Fractions CASE 1

$$\frac{P(x)}{Q(x)} = \frac{A_1}{a_1x + b_1} + \frac{A_2}{a_2x + b_2} + \dots \frac{A_q}{a_qx + b_q}$$

EXAMPLE

Evaluate $\displaystyle\int \frac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x}\,dx$.

Partial Fractions CASE 2: Repeated Linear Terms

When linear terms are repeated in the denominator $Q(x)$ then you need to repeat those terms in your expansion.

$$\frac{x^3 - x + 1}{x^2(x-1)^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{E}{(x-1)^3}$$

Partial Fractions CASE 3: Irreducible Quadratic Terms
If the denominator $Q(x)$ contains the expression $ax^2 + bx + c$ which can NOT be factored with real numbers (i.e. $b^2 - 4ac < 0$) then the expansion must contain $\dfrac{Ax + B}{ax^2 + bx + c}$

GroupWork

Write out the form of the partial fraction for the following expressions (and find the value of the unknown coefficients). If you have time, find the anti-derivative of the expression.

1. $\dfrac{2x^2 - x + 4}{x^3 + 4x} =$

2. $\dfrac{10}{5x^2 - 2x^3} =$

3. $\dfrac{x^6}{x^2 - 4} =$

4. $\dfrac{x^5 + 1}{(x^2 - x)(x^4 + 2x^2 + 1)} =$