Math 120 | **CALCULUS 2** | Spring 2014

Class 7: Wednesday February 5

Integration by Parts

GOAL

Similar to the way in which we obtain the method of **Integration by Substitution** from the differentiation rule called **The Chain Rule** we can obtain the method of **Integration by Parts** from the differentiation rule called **The Product Rule.**

Again, the goal is to exchange one (difficult) integral for another (less difficult) integral.

The Product Rule states that if u and v are both functions of x, then

$$(uv)' = \underline{\hspace{6cm}}.$$

If we integrate both sides of the equation above, we get

$$\underline{\hspace{2cm}} = \int (uv)'\ dx = \underline{\hspace{6cm}}.$$

Q: How is this useful?

A: It allows us to always exchange one integrand that consists of a product of functions for another integrad that is a product of functions. Hopefully, the second integrand will be simpler to anti-differentiate than the first one!

EXAMPLE

1. $\int 2xe^{7x}\ dx =$

2. $\int \ln(x)\ dx =$

NOTES

In **every** Integration By Parts problem we have exactly TWO choices. We look at the integrand and pick a function to ____________________ and one which to ____________________

Integration By Parts

In terms of functions $u(x)$ and $v(x)$ the Integration By Parts formula can be written as

$$\int u(x)v'(x)\ dx = u(x)v(x) - \int v(x)u'(x\ dx$$

OR (more simply)

$$\int u\ dv = uv - \int v\ du$$

General Rules for Integration By Parts
Generally you should choose to DIFFERENTIATE the MORE COMPLICATED FUNCTION in the integrand, and ANTI-DIFFERENTIATE the LESS COMPLICATED FUNCTION. This is because typically a function gets easier to handle when it gets differentiated, and this should make your integrand simpler.

List of Elementary Functions In Order Of Complexity

$$e^x \quad < \quad b^x \quad < \quad \sin(x) \quad < \quad \cos(x) \quad < \quad x^n \quad < \quad \ln(x)$$

In other words, the functions at the RIGHT END of the list are more likely to be the function you want to choose to DIFFERENTIATE and the functions at the LEFT END of the list are more likely to the functions you want to ANTI-DIFFERENTIATE

$\boxed{\text{GroupWork}}$

In **small groups of 3 or 4** use integration by parts to evaluate the following integrals:

3. $\int xe^{-x}\, dx =$

4. $\int x\cos(x)\, dx =$

5. $\displaystyle\int \frac{\ln(x)}{x^4}\ dx =$

6. $\displaystyle\int x^2 2^x\ dx =$

Using Integration By Parts on Definite Integrals

The general formula for applying integration by parts to a definite integral is

$$\int_a^b f(x)g'(x)\ dx = f(x)g(x)\ I\big|_a^b - \int_a^b g(x)f'(x)\ dx$$

$\boxed{\text{EXAMPLE}}$

7. $\displaystyle\int_0^1 \arctan(x)\ dx =$

8. $\displaystyle\int_1^2 (\ln(x))^2\ dx =$

GroupWork

Evaluate the following integrals

9. $\int_4^9 \frac{\ln(y)}{\sqrt{y}}\, dy =$

10. $\int_1^3 r^3 \ln(r)\, dr =$

11. $\int_1^4 t^{3/2} \ln(t)\, dt$

12. $\int_0^1 s e^{0.2s}\, ds =$