Class 5: Friday January 31

Practicing Integration

`Warm-Up`

Consider these following functions and compute their derivatives. Which ones do you have to use the chain rule on?

1. $f(x) = \sin(x^2)$, $\quad f'(x) = \dfrac{df}{dx} =$ ________________________

2. $g(x) = e^{\sin(x)}$, $\quad g'(x) = \dfrac{dg}{dx} =$ ________________________

3. $h(x) = \pi^x$, $\quad h'(x) = \dfrac{dh}{dx} =$ ________________________

Learning Techniques of Integration We want to learn some techniques for evaluating integrals, so we want to figure out how to find anti-derivatives. Not surprisingly, we can do so by recalling our rules of differentiation.

The Chain Rule: Derivative of a Composite Function

Suppose $h(x) = g(p(x))$. Then

$$\frac{dh}{dx} = h'(x) = g'(p(x)) \cdot p'(x)$$

The derivative of a composite function $h(x)$ is the product of the derivatives of the outside and inside functions. The derivative of the outside function must be evaluated at the inside function.

ANTI-Derivative of a Composite Function

We can use the basic idea behind the Chain Rule combined with the Fundamental Theorem of Calculus to evaluate the following integrals which appear to involve finding the anti-derivative of a composite function.

GROUPWORK

In each case try to identify the composite function while you evaluate the integral

1. $\displaystyle\int \sin(x^2)2x\, dx =$

2. $\displaystyle\int \cos(e^x)e^x\, dx =$

3. $\displaystyle\int \cos(x)[\sin(x)]^{23}\, dx =$

4. $\displaystyle\int u'(x)[u(x)]^{31}\, dx =$

5. $\displaystyle\int x^2(x^3+1)^{55}\, dx =$

All of these integrals follow a pattern in the appearance of the integrand.

The integral looks like:

Integration By Substitution

Let's try to regularize the process of integrals which fit the pattern of the ones on the previous page into a *technique of integration* called **integration by substitution**.

$\int (x + e^x)^9 (1 + e^x)\, dx =$

STEP 1: Identify u-substitution

Let $u =$

STEP 2: Compute $u'(x)$

$\frac{du}{dx} =$

Multiplying both sides by dx gives: $du =$

STEP 3: Translate integral from original into u

Now substitute into the original integral, so that everything is in terms of u instead of x.

$\int (x + e^x)^9 (1 + e^x)\, dx = \int$

This new integral in u-variables should be easier to evaluate than the previous one in x-variables.

STEP 4: Evaluate integral in u-variables:

STEP 5: Translate final answer back into original variable

Now "convert back to x"

Exercise

$\int \frac{t^2}{5 + t^3}\, dt =$

Let $u =$

Then $\frac{du}{dt} =$

So $du =$

So $(\quad) \cdot du = t^2\, dt$

Substitute, then solve:

$\int \frac{t^2}{5 + t^3}\, dt =$

Convert back into the original t variable:

EXAMPLE

How does the integration by substitution technique change when you have a DEFINITE INTEGRAL?

$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx =$