Class 4: Wednesday January 29

The Fundamental Theorem of Calculus

Warm-Up

Use the Fundamental Theorem of Calculus (FTC) to evaluate the following integrals:

1. $\int_1^{-1} x^3\ dx =$
2. $\int_0^{\pi} \sin(x)\ dx =$
3. $\int_2^{5} e^{x^2}\ dx =$
4. $\int_0^{1} \sqrt[3]{3x}\ dx =$

Differentiating Integrals

The first part of the FTC is really about anti-differentiating derivatives, while the second part of the FTC is about differentiating anti-derivatives. (Note: your textbook arranges the parts of the FTC in different order.) The central point of the FTC is to see differentiation and integration as **inverse processes** of each other. Another key point is to realize these processes act on functions as their input.

Theorem

The Fundamental Theorem of Calculus (Part Two): If $f(t)$ is continuous on $[a, b]$ and $F(x)$ is defined on $[a, b]$ as

$$F(x) = \int_a^x f(t)\ dt$$

then $F'(x) = f(x)$ on (a, b). In other words, $\frac{d}{dx}\int_a^x f(t)\ dt = f(x)$.

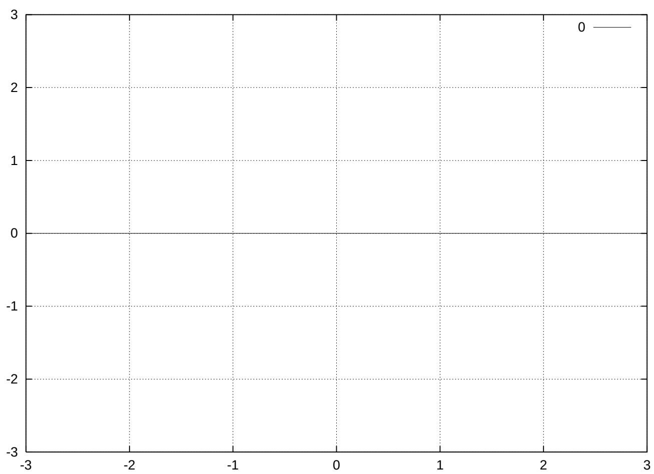
EXAMPLE

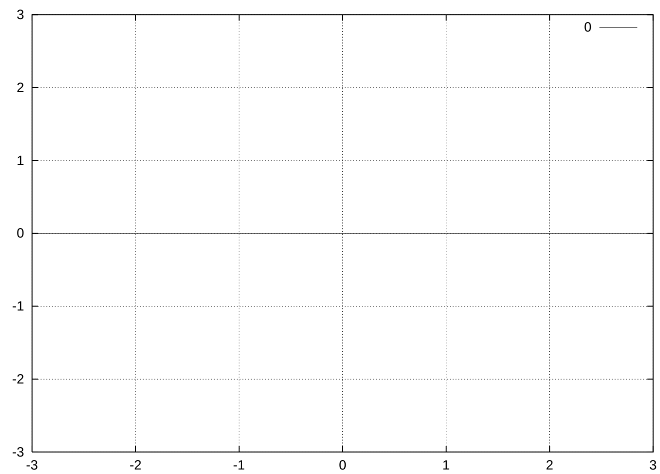
Let's think about what the expression $\int_a^x f(t)\ dt$ means. This is sometime called an **accumulation function**.

What quantity about the function $f(t)$ is being accumulated?: ____________________

Let's try and visualize what an accumulation function looks like.

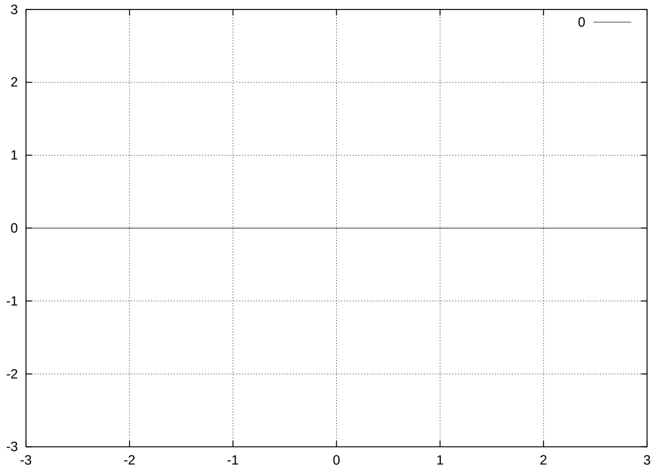
As x varies, what does the graph of the accumulation function $A(x) = \int_0^x f(t)\ dt$ of the constant function $f(t) = C$ from 0 to x look like? Sketch it below (to the right). What is $A(0)$? $A(1)$? $A(-1)$? $A(2)$?

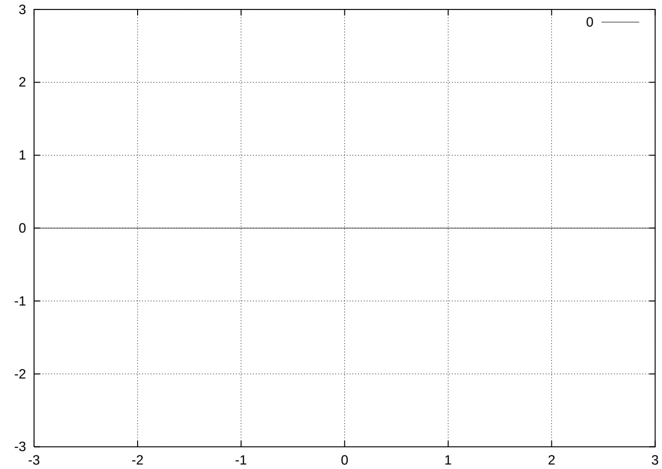




Exercise

Now, let us define another accumulation function $B(x)$ for the *linear* function $f(t) = t$ as $B(x) = \int_0^x t\ dt =$. I want you to sketch a graph of B on the axes below (to the right).





What happens if you start accumulating from a different point?
Draw a graph of the function $C(x) = \int_1^x t\ dt$ on the same axes you sketched $B(x)$.

GroupWork

Consider the following accumulation functions. Use the FTC to find the derivative of each.

1. $F(x) = \int_0^x e^{-s}\ ds$. What is $F'(x)$?

2. $G(x) = \int_x^2 \ln(t^2 + 1)\ dt$. What is $G'(x)$?

3. $H(x) = \int_1^{x^2} e^t\ dt$. What is $H'(x)$?

Average Value Of A Function

We can define the average value of a function f_{ave} over an interval $[a, b]$ as

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x)\ dx$$

which means that if $f(x)$ is continuous on $[a, b]$ then there always exists a number c such that

$$f(c) \cdot (b - a) = \int_a^b f(x)\ dx$$

This result is known as the **Mean Value Theorem For Integrals**.