

SHOW YOUR WORK

Find the interval of convergence and radius of convergence for each of the following infinite series

(a) (5 points) $\sum_{k=1}^{\infty} -\frac{x^{2k}}{k} = -x^2 - \frac{x^4}{2} - \frac{x^6}{3} + \frac{x^4}{4} - \dots$

$$\frac{1}{R} = \lim_{k \rightarrow \infty} \left| \frac{-\frac{1}{k+1}}{-\frac{1}{k}} \right| = \lim_{k \rightarrow \infty} \frac{k}{k+1} = 1$$

Check

$$x = 1$$

$$\sum_{k=1}^{\infty} -\frac{1}{k} 1^{2k} = \sum_{k=1}^{\infty} -\frac{1}{k} \quad \text{DIVERGES}$$

Check

$$x = -1$$

$$\sum_{k=1}^{\infty} -\frac{1}{k} (-1)^{2k} = \sum_{k=1}^{\infty} -\frac{1}{k} \quad \text{DIVERGES}$$

Interval
of
convergence
 $(-1, 1)$

$$R = 1$$

(b) (5 points) $\sum_{k=0}^{\infty} \frac{x^k}{k!}$

$$\frac{1}{R} = \lim_{k \rightarrow \infty} \left| \frac{\frac{1}{(k+1)!}}{\frac{1}{k!}} \right| = \lim_{k \rightarrow \infty} \frac{k!}{(k+1)!} = \lim_{k \rightarrow \infty} \frac{1}{k+1} = 0 \quad R = \infty$$

Converges for every x value, i.e. $(-\infty, \infty)$