Math 120 Spring 2014 Quiz **SIX**

SHOW YOUR WORK

Consider the following initial value problem (IVP)

$$\frac{dy}{dx} = xe^y, \quad y(0) = 0$$

(a) (*6 points*) Show that the solution to the IVP is $y(x) = -\ln(1 - \frac{x^2}{2})$ by using the method of separation of variables.

$$\frac{dy}{dx} = xe^y$$

$$\frac{dy}{e^y} = x\,dx$$

$$\int e^{-y}dy = \int x\,dx$$

$$-e^{-y} = \frac{x^2}{2} + C$$

$$e^{-y} = -\frac{x^2}{2} - C$$

$x = 0, y = 0$ $\quad e^0 = -\frac{0^2}{2} - C \Rightarrow -C = 1 \Rightarrow C = -1$

$$e^{-y} = -\frac{x^2}{2} + 1$$

$$-y = \ln\left(-\frac{x^2}{2} + 1\right)$$

$$y = -\ln\left(1 - \frac{x^2}{2}\right)$$

(b) (*4 points*) Confirm that the given function does indeed satisfy the initial value problem (i.e. the differential equation AND initial condition).

I.C. $x = 0, y = 0$?

$x = 0$, $y = -\ln\left(1 - \frac{0^2}{2}\right) = -\ln(1) = 0$ ✓

DE. $y' = xe^y$

$$y' = \left[\ln\left(1 - \frac{x^2}{2}\right)\right]' = -\frac{1}{1 - \frac{x^2}{2}} \cdot \left(1 - \frac{x^2}{2}\right)'$$

$$= -\frac{1}{1 - \frac{x^2}{2}} \cdot (-x)$$

$$= \frac{1}{1 - \frac{x^2}{2}} \cdot x = \frac{1}{e^{-y}} \cdot x = e^y \cdot x$$ ✓