Math 120 Spring 2014 Quiz **FIVE**

SHOW YOUR WORK

Consider the improper integrals below. Write down whether you think the integral **converges** or **diverges** in the box (1 point). By selecting an appropriate integral (1 point), prove your selected choice is correct by using the comparison theorem (3 points).

(a) (*5 points*) $\mathcal{J} = \int_1^\infty e^{t^2+t+1}\,dt$ $\boxed{\text{DIVERGES}}$

We need to show $\mathcal{J} >$ a divergent integral.

We know $\int_1^\infty e^t$ diverges $\qquad \int_1^\infty e^t\,dt = \lim_{b\to\infty} \int_1^b e^t\,dt = \lim_{b\to\infty} e^b - e^1 = \infty$

$t > 1$

$t^2 > t$

$t^2 + t > t$

$t^2 + t + 1 > t$

$e^{t^2+t+1} > e^t$

$\int_1^\infty e^{t^2+t+1}\,dt > \int_1^\infty e^t\,dt$ ~~Since~~ So $\mathcal{J}$ diverges by comparison to $\int_1^\infty e^t\,dt$

(b) (*5 points*) $\mathcal{K} = \int_1^\infty \frac{1}{\sqrt{s^4+1}}\,ds$ $\boxed{\text{CONVERGES}}$

As $s \to \infty$ $\frac{1}{\sqrt{s^4+1}} \sim \frac{1}{\sqrt{s^4}} = \frac{1}{s^2}$ which we know $\int_1^\infty \frac{1}{s^2}\,ds$ CONVER

We need to show $I <$ a convergent integral

$s > 1$

$s^2 > 1$

$s^4 > s^2$

$s^4 + 1 > s^4$

$\frac{1}{s^4+1} < \frac{1}{s^4}$

$\frac{1}{\sqrt{s^4+1}} < \frac{1}{\sqrt{s^4}}$ $\Rightarrow \frac{1}{\sqrt{s^4+1}} < \frac{1}{s^2}$

$\int_1^\infty \frac{1}{\sqrt{s^4+1}}\,ds < \int_1^\infty \frac{1}{s^2}\,ds$

We know $\int_1^\infty \frac{1}{s^2}\,ds$ CON $\quad p = 2 > 1 \quad \int_1^\infty \frac{dx}{x^p}$ rule

So, by comparison $\int_1^\infty \frac{1}{\sqrt{s^4+1}}\,ds$ converges too