Math 120 Spring 2014 **BONUS Quiz FOUR**

SHOW ALL YOUR WORK!

Math 120 Spring 1996 Final Exam, Question #2.

Use the N^{th} Term test, the Basic Comparison Test or the Limit Comparison Test to determine whether or not the following infinite series converge or not.

(i) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{2^k}{3^k}\right)\frac{1}{k^4}$ This looks like it converges

Use Limit Comparison Theorem.

$$\lim_{K\to\infty}\left|\frac{\left(\frac{2}{3}\right)^K\frac{1}{K^4}}{\frac{1}{K^4}}\right| = \lim_{K\to\infty}\left(\frac{2}{3}\right)^K = 0$$

Converges by Limit Comparison Test

(ii) $\displaystyle\sum_{k=1}^{\infty}\cos(2\pi k)$ $= \cos(2\pi) + \cos(4\pi) + \cos(6\pi) + \ldots$

$= 1 + 1 + 1 + \ldots$

DIVERGES by n^{th} Term test

(iii) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{k^2+3}{k^2}\right)\frac{1}{k^2}$ $= \displaystyle\sum_{K=1}^{\infty}\frac{K^2+3}{K^4}$

As $K\to\infty$ $\frac{K^2+3}{K^4} \sim \frac{1}{K^2}$

$\frac{K^2+3}{K^4} < \frac{2}{K^2}$

$K^4+3K^2 < 2K^4$

$K^4+3K^2 < K^4+K^4$

$3K^2 < K^4$

$3 < K^2$

This is true for $K>2$

So, by Basic Comparison Test $\displaystyle\sum_{K=1}^{\infty}\frac{K^2+3}{K^4} < \sum_{K=1}^{\infty}\frac{2}{K^2}$

and we know $\sum\frac{2}{K^2}$ converges (p-series)

so $\sum\frac{K^2+3}{K^4}$ converges