Math 120 Spring 2014 **BONUS Quiz ONE**

SHOW ALL YOUR WORK AND EXPLAIN EVERY ANSWER

Given the following information about an unknown function $g(x)$ which is always continuous and differentiable

$$\int_1^2 \frac{g(u)}{u^2}\,du = 3, \quad \int_{\frac{1}{2}}^2 \frac{g(u)}{u^2}\,du = 5, \quad g(1/2) = 2, \quad g(1) = -2, \quad g(2) = 1, \quad g(4) = 4$$

(a) [*5 points*] Evaluate $I = \displaystyle\int_1^2 g\left(\frac{1}{x}\right)\,dx$. [HINT: Use integration by substitution].

IBS

$u = \frac{1}{x}$

$du = -\frac{1}{x^2}dx$

$-x^2 du = dx$

$-\left(\frac{1}{u}\right)^2 du = dx$

$x = 1,\ u = 1$

$x = 2,\ u = \frac{1}{2}$

Note:

$$\int_{1/2}^{1} \frac{g(u)\,du}{u^2} = \int_{1/2}^{2} \frac{g(u)}{u^2}du + \int_{2}^{1} \frac{g(u)\,du}{u^2}$$

$$I = \int_1^{1/2} -g(u)\frac{1}{u^2}\,du = -\int_1^{1/2} \frac{g(u)\,du}{u^2} = \int_{1/2}^{1} \frac{g(u)\,du}{u^2} = \int_{\frac{1}{2}}^{2} \frac{g(u)\,du}{u^2} - \int_1^2 \frac{g(u)\,du}{u^2}$$

$$I = 5 - 3 = \boxed{2}$$

(b) [*5 points*] Evaluate $J = \displaystyle\int_{1/2}^2 \frac{g'(x)}{x}\,dx$. [HINT: Use integration by parts].

IBP

$u = \frac{1}{x} \qquad u' = -\frac{1}{x^2}$

$v' = g'(x) \qquad v = g(x)$

$$J = \frac{1}{x}g(x)\Big|_{1/2}^{2} - \int_{1/2}^{2} -\frac{1}{x^2}g(x)\,dx$$

$$= \frac{1}{2}g(2) - \frac{1}{1/2}g(1/2) + \int_{1/2}^{2} \frac{g(x)}{x^2}dx$$

$$= \frac{1}{2}\cdot 1 - 2\cdot 2 + 5$$

$$\boxed{J = 3/2}$$