Using Fourier Series
Class 35: Friday April 25

Fourier Series
Taylor used polynomials to approximate functions.
Fourier used trigonometric functions to approximate **periodic** functions.

We write $P_n(x)$ for the nth degree Taylor polynomial.
Example: $P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$.

We write $F_n(x)$ for the nth degree Fourier "polynomial".
Example: $F_3(x) = a_0 + a_1\cos(x) + a_2\cos(2x) + a_3\cos(3x) + b_1\sin(x) + b_2\sin(2x) + b_3\sin(3x)$.

a_k and b_k are some constants. They are called the <u>coefficients</u>.

A Taylor Series is $\displaystyle\sum_{k=1}^{\infty} a_k x^k$. A Fourier Series is $a_0 + \sum_{k=1}^{\infty} b_k\sin(kx) + a_k\cos(kx)$

For a periodic function $f(t)$ whose period is 2π, the coefficients of its Fourier Series are:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)dt \qquad a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(kt)dt,\ k = 1, 2, 3, \cdots$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(kt)dt,\ k = 1, 2, 3, \cdots$$

EXAMPLE

$$f(x) = \begin{cases} 7 & \text{if } (2n)\pi \le x \le (2n+1)\pi \\ 0 & \text{if } (2n+1)\pi < x < (2n+2)\pi \end{cases}$$

1. Sketch the graph of $f(x)$ below.

2. Find the first degree Fourier polynomial for $f(x)$.

3. Find the second degree Fourier polynomial for $f(x)$.