Intervals of Convergence of Power Series
Class 33: Monday April 21

Interval of Convergence: Given a power series

$$\sum_{k=1}^{\infty} b_k (x-a)^k,$$

we want to find the set of x-values for which the series converges. This set is called the **interval of convergence**. The interval of convergence is always centered on the point a. To compute the interval of convergence, apply the **Absolute Ratio Test** to the power series.

Examples

$$\sum_{k=0}^{\infty} x^n$$

$$\sum_{k=1}^{\infty} \frac{x^n}{n}$$

$$\sum_{k=0}^{\infty} (n+1)x^n$$

$$\sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

$$\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

There are two types of intervals:

1. All Real Numbers. (Infinite Radius of Convergence)

2. A finite interval of the form $(a - R, a + R)$, $[a - R, a + R]$, $(a - R, a + R]$ or $[a - R, a + R)$. R is called the *radius of convergence.*

GroupWork Find the interval of convergence of each of the following power series:

1. $\displaystyle\sum_{k=0}^{\infty} k^2 e^{-k} x^k$

2. $\displaystyle\sum_{k=1}^{\infty} \frac{x^k}{k^2}$

3. $\displaystyle\sum_{k=0}^{\infty} k! x^k$

4. $\displaystyle\sum_{k=0}^{\infty} \frac{2k^2 + 1}{k^2 - 5}(x - 2)^n$

5. $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{(x - 5)^k}{k \ln(k)}$

6. $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{(x - 2)^k}{3^k}$