Applications of Taylor Polynomials
Class 30: Monday April 14

You should be able to use the derivative definition of a Taylor Series to compute the Taylor Series of these "popular" functions about $x = 0$ given below.

$$\begin{aligned}
\sin(x) &= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots &&= \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} \\
\cos(x) &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots &&= \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} \\
\frac{1}{1-x} &= 1 + x + x^2 + x^3 + \dots &&= \sum_{k=0}^{\infty} x^k \\
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} + \dots &&= \sum_{k=1}^{\infty} (-1)^{(k+1)} \frac{x^k}{k} \\
e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots &&= \sum_{k=0}^{\infty} \frac{x^k}{k!}
\end{aligned}$$

Getting One Taylor Series From Another

Notice that if you write out the terms of the Taylor Series for $\cos(x)$ about $a = 0$ you can notice that it can be obtained from the Taylor Series for $\sin(x)$ about $a = 0$.

Similarly, we could obtain the Taylor Series for $f(x) = \dfrac{1}{1+x^2}$ and use this information to obtain a Taylor Series for $\arctan(x)$.

Use the previous information to approximate $\int_0^1 \frac{1}{1+x^2} dx$

How about $\int_0^1 \frac{1}{1+x^3} dx$?

Find an approximate value of $\int_0^1 e^{\sin t}\, dt$