Alternating Series and more Convergence Tests
Class 27: Monday April 7

What do the following series have in common?

$1 - 2 + 3 - 4 + 5 - 6 + \cdots \qquad 1/1 - 1/2 + 1/3 - 1/4 + \cdots \qquad -1/1^2 + 1/2^2 - 1/3^2 + 1/4^2 - \cdots$

Series of this type are called ____________________________.

1. Which of the following series is alternating?

(a) $\displaystyle\sum_{k=4}^{\infty}(-1)^k \frac{3}{k}$

(b) $\displaystyle\sum_{k=4}^{\infty}(-1)^{2k} \frac{3}{k}$

(c) $\displaystyle\sum_{k=4}^{\infty}(-1)^{k+1} \frac{3}{\ln(k)}$

(d) $\displaystyle\sum_{k=0}^{\infty}(-1)^k \frac{k}{1000}$

(e) $\displaystyle\sum_{k=0}^{\infty}(-1)^k \frac{k+10}{3k+1}$

Theorem

If in an alternating series, $a_1 - a_2 + a_3 - a_4 + \cdots$, both of the following conditions hold, then the series converges.

1. Each term has smaller magnitude than the previous one, i.e., $a_{k+1} < a_k$.
2. $\lim_{k\to\infty} a_k = 0$.

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2. For each of the series in (1) that is alternating, determine which, if any, of the two conditions ofthe theorem hold. Which series converge, and which diverge?

Root Test

For an infinite series $\sum a_n$ where $a_n \geq 0$

$$L = \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} (a_n)^{1/n}$$

Then

If $L < 1$, the series converges.

If $L > 1$, the series diverges.

If $L = 1$, the series may converge or diverge. The root test provides no information. (Use another test!)

GroupWork Let's use the root test on the following series to tests for convergence

$$\sum_{k=1}^{\infty} \frac{1}{k^k}$$

$$\sum_{k=0}^{\infty} \frac{k^k}{e^k}$$

$$\sum_{k=0}^{\infty} e^{-k^2}$$